Consecutive terms of a **geometric sequence** have a common ratio. We call the **common ratio** r.

$$a_1, a_2, a_3, a_4, \ldots, a_n, \ldots$$

$$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = r \quad , r \neq 0$$

Determine whether the sequence is geometric. If so, find the common ratio.

$-2, 1, 4, 7, \ldots$

$\frac{1}{-2} = \frac{4}{1} = ?$ NO (arithmetic)

$1, 1, 2, 3, 5, 8, \ldots$

$\frac{1}{1} = \frac{2}{1} ?$ NO

$1, 3, 6, 9,$

$\frac{3}{1} = \frac{6}{3} ?$ NO

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$ yes

$\frac{\frac{1}{2}}{1} = \frac{\frac{1}{4}}{\frac{1}{2}} ?$ $r = \frac{1}{2}$

$\frac{1}{4} \cdot \frac{2}{1} = \frac{1}{2}$

$1, 4, 9, 16, \ldots$

$\frac{4}{1} = \frac{9}{4} ?$ NO

$3, 9, 27, 81, \ldots$

$\frac{9}{3} = \frac{27}{9} = \frac{81}{27} = 3$ Yes $r = 3$

Example:

1. Write the first four terms of the geometric sequence whose nth term is $6(-2)^n$. Then find the common ratio of the consecutive terms.

$a_1 = 6(-2)^1 = -12$

$a_2 = 6(-2)^2 = 24$

$a_3 = 6(-2)^3 = -48$

$a_4 = 6(-2)^4 = 96$

$\frac{24}{-12} = -2$

$\frac{-48}{24} = -2$

$\frac{96}{-48} = -2$

$r = -2$

The *n*th Term of a Geometric Sequence:

The nth term of a geometric sequence has the form $a_n = a_1 r^{n-1}$ where r is the common ratio of consecutive terms of the sequence and a_1 is the first tem.

Example:

2. Write the first five terms of the geometric sequence whose first term is $a_1 = 2$ and whose common ratio is $r = 4$.

$\boxed{2, 8, 32, 128, 512}$

$a_1 = 2$

$a_2 = 2(4)^1$

$a_3 = 2(4)^2$

$a_4 = 2(4)^3$

$a_5 = 2(4)^4$

Example:

3. Find the 12th term of the geometric sequence whose first term is 14 and whose common ratio is 1.2 .

$a_1 = 14$

$r = 1.2$

$n = 12$

$a_n = 14(1.2)^{n-1}$

$a_{12} = 14(1.2)^{11}$

$\boxed{a_{12} = 104.02}$

4. Find the 12^{th} term of the geometric sequence 4, 20, 100, ...

$a_1 = 4$

$r = \frac{20}{4} = 5$

$a_n = 4(5)^{n-1}$

$a_{12} = 4(5)^{11}$

$\boxed{a_{12} = 195{,}312{,}500}$

5. The second term of a geometric sequence is 6, and the fifth term is 81/4. Find the eighth term. (Assume that the terms of the sequence are positive.)

$a_2 = 6$

$a_5 = \frac{81}{4}$

$\_\ \underline{6}\ \_\ \_\ \frac{81}{4}$

$5-2$

$6 \cdot r \cdot r \cdot r = \frac{81}{4}$

$\frac{1}{6} \cdot 6r^3 = \frac{81}{4} \cdot \frac{1}{6}$

$\sqrt[3]{r^3} = \sqrt[3]{\frac{27}{8}}$

$r = \frac{3}{2}$

$\frac{6}{a_1} = \frac{3}{2}$

$\frac{6}{\frac{3}{2}} = a_1$

$6 \cdot \frac{2}{3} = 4 = a_1$

$a_n = 4\left(\frac{3}{2}\right)^{n-1}$

$a_8 = 4\left(\frac{3}{2}\right)^7$

$= \boxed{\frac{2187}{32}}$

The Sum of a Finite Geometric Sequence

$r \neq 1$

$$S_n = \sum_{i=1}^{n} a_1 (r)^{i-1} = a_1\left(\frac{1-r^n}{1-r}\right)$$

Example:

5. Find the sum $\sum_{i=1}^{10} 2(0.25)^{i-1}$.

sum of 1^{st} 10 terms

$a_1 = 2(0.25)^{1-1}$
$= 2(.25)^0$
$= 2(1)$
$= 2$

$S_{10} = 2\left(\frac{1-(0.25)^{10}}{1-0.25}\right)$

$= 2.667$

Geometric Series

The summation of the terms of an infinite geometric *sequence* is called an infinite geometric series.

The Sum of an Infinite Geometric Series

If $|r| < 1$

Then $S_n = \sum_{i=0}^{\infty} a_1 r^i = \frac{a_1}{1-r}$ ← 1st term

Example:

7. Find each sum.

a) $\sum_{n=0}^{\infty} 5(0.5)^n$ ← r

$S_\infty = \frac{5}{1-0.5}$

$= \frac{5}{0.5}$

$= \boxed{10}$

b) $5 + 1 + 0.2 + 0.04 + \ldots$

$r = \frac{1}{5} = 0.2$

$a_1 = 5$

$S_\infty = \frac{5}{1-0.2}$

$= \frac{5}{0.8} = \boxed{6.25}$

8. An investor deposits \$70 on the first day of each month in an account that pays 2% interest, compounded monthly. What is the balance at the end of 4 years?

48 months

$A_{48} = 70\left(1 + \frac{0.02}{12}\right)^{48} = 70(1.001\overline{6})^{48}$

$A_{47} = 70\left(1 + \frac{0.02}{12}\right)^{47} = 70(1.001\overline{6})^{47}$

$\vdots$

$A_1 = 70\left(1 + \frac{0.02}{12}\right)^{1} = 70(1.001\overline{6})^{1}$

$a_1 = 70(1.001\overline{6})$

$S = a_1\left(\frac{1-r^n}{1-r}\right)$

$S_{48} = 70\underbrace{\left(1 + \frac{.02}{12}\right)^{1}}_{x}\left(\frac{1-\left(1+\frac{0.02}{12}\right)^{48}}{1-\left(1+\frac{0.02}{12}\right)}\right)$

$x = 1.001\overline{6}$

$= 70x\left(\left(1-x^{48}\right)/\left(1-x\right)\right) = \boxed{\$3500.85}$

p. 631: 5, 9, 11, 13, 17, 25, 31, 33, 38, 41, 45, 55, 61, 67, 69, 71, 73, 75, 79, 85, 91