

A sequence whose consecutive terms have a common difference between consecutive terms is called an arithmetic sequence. We call the **common difference** d .

$$a_1, a_2, a_3, a_4, \dots, a_n, \dots$$

$$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \dots = d$$

Determine whether the sequence is arithmetic. If so, find the common difference.

$$-2, 1, 4, 7, \dots \text{ yes } d=3 \quad 1, 1, 2, 3, 5, 8, \dots \text{ NO}$$

$$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots \quad 1, 4, 9, 16, 25, \dots \text{ NO}$$

$-\frac{1}{2} \quad -\frac{1}{4}$
 NO
 (geometric)

Example:

1. Write the first four terms of the arithmetic sequence whose n th term is $3n-1$. Then find the common difference between consecutive terms.

$$\begin{aligned}
 a_1 &= 3(1) - 1 = 2 & 5 - 2 &= 3 \\
 a_2 &= 3(2) - 1 = 5 & 8 - 5 &= 3 & d &= 3 \\
 a_3 &= 3(3) - 1 = 8 \\
 a_4 &= 3(4) - 1 = 11 & 11 - 8 &= 3
 \end{aligned}$$

1, 4, 9, 16, ... Not arithmetic b/c $9 - 4 \neq 4 - 1$

$$\begin{aligned}
 a_1 &= a_1 & 1^{\text{st}} \\
 a_2 &= a_1 + d & 2^{\text{nd}} \\
 a_3 &= a_1 + 2d & 3^{\text{rd}} \\
 a_4 &= a_1 + 3d & 4^{\text{th}} \\
 a_n &= a_1 + (n-1)d
 \end{aligned}$$

$$a_n = a_1 + (n-1)d$$

\uparrow 1st Term \uparrow common difference

The n th Term of an Arithmetic Sequence:

The n th term of an arithmetic sequence has the form $a_n = a_1 + (n-1)d$ where d is the common difference between consecutive terms of the sequence and a_1 is the first term.

Example:

2. Find a formula for the n th term of the arithmetic sequence whose common difference is 5 and whose first term is -1.

$$a_1 = -1 \quad d = 5$$

$$\begin{aligned}
 a_n &= -1 + (n-1)(5) \\
 \text{OR} \quad a_n &= -1 + 5(n-1)
 \end{aligned}$$

Partial Sum of an Arithmetic Sequence

Example

7. Find the 120th partial sum of the arithmetic sequence 6, 12, 18, 24, 30, ...

$$\begin{aligned} n=120 \quad S_{120} &= \frac{120}{2}(6+720) && \begin{aligned} a_1 &= 6 & d &= 6 \\ a_n &= 6 + (n-1)(6) \\ a_{120} &= 6 + (120-1)(6) \\ a_{120} &= 720 \end{aligned} \\ &= 60(726) \\ &= \boxed{43,560} \end{aligned}$$

8. Find the 30th partial sum of the arithmetic sequence 78, 76, 74, 72, 70, ...

$$\begin{aligned} n=30 \quad S_{30} &= \frac{30}{2}(78+20) && \begin{aligned} a_1 &= 78 & d &= -2 \\ a_n &= 78 + (n-1)(-2) \\ a_{30} &= 78 + (30-1)(-2) \\ a_{30} &= 20 \end{aligned} \\ &= \boxed{1470} \end{aligned}$$

9. A company sells \$160,000 worth of printing paper during its first year. The sales manager has set a goal of increasing annual sales of printing paper by \$20,000 each year for 9 years. Assuming that this goal is met, find the total sales of printing paper during the first 10 years this company is in operation.

$$\begin{aligned} a_1 &= 160,000 \\ d &= 20,000 \\ n-1 &= 9 & a_{10} &= 160,000 + 9(20,000) \\ n &= 10 & &= 340,000 \end{aligned} \quad S_{10} = \frac{10}{2}(160,000 + 340,000) = 5(500,000) = \boxed{\$2,500,000}$$