Infinite Sequence: A function whose domain is the set of positive integers. The function values $a_1, a_2, a_3, a_4, ..., a_n, ...$ are the teams of the sequence. When the domain of the function consists of the first n positive integers only, the sequence is a finite sequence.

Example: Write the first four terms of the sequence given by the function.

1.
$$a_n = 2n+1$$

 $O_1 = 2(1)+1 = 3$
 $O_2 = 2(2)+1 = 5$
 $O_3 = 2(3)+1 = 7$
 $O_4 = 2(4)+1 = 9$

$$2. a_{n} = \frac{2 + (-1)^{n}}{n}$$

$$Q_{1} = \frac{2 + (-1)^{1}}{1} = \frac{1}{1} = 1$$

$$Q_{2} = \frac{2 + (-1)^{2}}{2} = \frac{3}{2}$$

$$Q_{3} = \frac{2 + (-1)^{3}}{3} = \frac{1}{3}$$

$$Q_{4} = \frac{2 + (-1)^{4}}{4} = \frac{3}{4}$$

$$a_3 = \frac{2 + (-1)^3}{3} = \frac{1}{3}$$

$$a_4 = \frac{2 + (-1)^4}{4} = \frac{3}{4}$$

The nth term must be given to define a unique sequence.

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{15}, \dots, a_n \dots$$

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots, a_n \dots$$

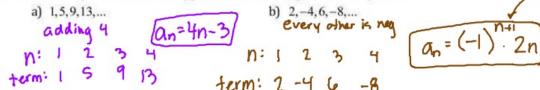
$$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots, a_n \dots$$

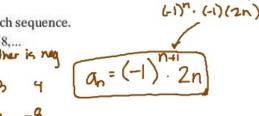
$$\frac{1}{2} = \frac{1}{2}$$

Finding the nth term of a Sequence

Look for an apparent pattern for the sequence, then write the expression for the *n*th term. $(-1)^n \cdot (-2n)^n$

Write an expression for that apparent nth term (a_e) of each sequence.





Recursive Sequence: To define a sequence recursively, you need to be given one or more of the first few terms. All other terms of the sequence are then defined using previous terms.

Example:

Write the first five terms of the sequence defined recursively as a₁ = 6, a_{k+1} = a_k + 1, where k ≥ 2.

$$a_1 = 6$$
 $a_2 = (e + 1 = 7)$
 $a_2 = a_{k+1} = a_1 + 1$
 $a_3 = 7 + 1 = 8$
 $a_4 = 8 + 1 = 9$
 $a_3 = a_{2+1} = a_2 + 1$
 $a_3 = a_{2+1} = a_2 + 1$
 $a_4 = 9 + 1 = 10$
 $a_5 = 9 + 1 = 10$

Sometimes it is convenient to begin subscripting a sequence with 0 instead of 1 so that the terms of the sequence become $a_0, a_1, a_2, a_3, a_4, \dots$ When this is the case, the domain includes 0.

$$a_2 = a_{2-2} + a_{2-1}$$

 $a_0 + a_1$

$$A_{4} = A_{4-2} + A_{4+1}$$

$$= A_{2} + A_{3}$$

$$= 2 + 3 = 5$$

 $Q_2 = Q_{2-2} + Q_{2-1}$ $Q_0 + Q_1$ $Q_1 + Q_2$ $Q_2 + Q_3$ $Q_1 + Q_4$ $Q_1 + Q_4$ $Q_1 + Q_4$ $Q_2 + Q_3$ $Q_1 + Q_4$ $Q_2 + Q_4$ $Q_3 + Q_4$ $Q_4 + Q_4$ $Q_5 + Q_6$ $Q_7 + Q_8$ $Q_8 + Q_8$ $Q_$ Example:

$$a_0 = 1$$
, $a_1 = 3$, $a_k = a_{k-2} + a_{k-1}$, where $k \ge 2$.

$$G_3 = A_{3-2} + A_{3-2}$$
= $A_1 + A_2$
= 3+4
= 7

Summation Notation: The sum of the first n terms of a sequence is represented by

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + a_4 + \dots + a_n$$

where is called the index of summation, nisthe upper limit of summation, and 1 is the lower limit of summation.

upper and I is the rough.

limit $\sum_{c=1}^{\infty}$ means to find the sum. c=1 lower limit $\sum_{c=1}^{\infty}$ shorthand for adding terms of a sequence

n is positive

Example:

8. Find the sum
$$\sum_{i=1}^{4} (4i+1)$$
.

$$= \left[\frac{4}{1} (1)+1 \right] + \left[\frac{4}{2} (2)+1 \right] + \left[\frac{4}{3} (3)+1 \right] + \left[\frac{4}{3} (4)+1 \right]$$

$$= 5 + 9 + 13 + 17 = \boxed{44}$$

Properties of Sums:

1.
$$\sum_{i=1}^{n} c = cn$$

erties of Sums:
1.
$$\sum_{i=1}^{n} c = cn$$
 $\sum_{i=1}^{7} 4 + 7 = 28$ 2. $\sum_{i=1}^{n} ca_{i} = c \sum_{i=1}^{n} a_{i}$

3.
$$\sum_{i=1}^{n} a_i + b_i = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$$
 4. $\sum_{i=1}^{n} a_i - b_i = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i$

4.
$$\sum_{i=1}^{n} a_i - b_i = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i$$

Sequence: list of # series: sum of #

Series: The sum of a series.

Consider the infinite sequence $a_1, a_2, a_3, ..., a_i, ...$

1. The sum of the first n terms of the sequence is called a finite series or the nth partial sum of the sequence and is denoted by

$$\frac{a_1 + a_2 + a_3 + \cdots + a_n}{a_1 + a_2 + a_3 + \cdots + a_n} = \sum_{i=1}^n a_i$$

2. The sum of all the terms of the infinite sequence is called an <u>infinite series</u> and is denoted by

$$a_1 + a_2 + a_3 + \dots + a_i + \dots = \sum_{i=1}^{\infty} a_i$$

Example:

- 9. For the series $\sum_{i=0}^{\infty} \frac{5}{10^i}$ find:
 - a) the fourth partial sum

$$\sum_{i=1}^{4} \frac{5}{10^{i}} = \frac{5}{10^{1}} + \frac{5}{10^{2}} + \frac{5}{10^{4}} + \frac{5}{10^{4}}$$

$$= .5 + .05 + .005 + .0005$$

$$= (.5555)$$

$$\sum_{i=1}^{8} \frac{5}{10^{i}} = \frac{5}{10^{i}} + \frac{5}{10^{n}} + \frac{5}{10^{n}} + \frac{5}{10^{n}} + \cdots$$

$$= .555555555$$

$$= \frac{5}{10^{n}}$$

 An investor deposits \$1000 in an account that earns 3% interest compounded monthly. The balance in the account after n months is given by

$$A_n = 1000(1 + \frac{0.03}{12})^n$$
, $n = 0, 1, 2, ...$

- a) Write the first three terms of the sequence.

b) Find the balance in the account after four years by computing the 48th term of the sequence.

a)
$$A_0 = 1000 \left(1 + \frac{0.03}{12}\right)^0 = 1000$$

$$A_1 = 1000 \left(1 + \frac{0.03}{12}\right)^1 = 1002.50$$

$$A_2 = 1000 \left(1 + \frac{0.03}{12}\right)^2 = 1005.01$$

= \$1127.33