**Infinite Sequence:** A function whose domain is the set of positive integers. The function values $a_1, a_2, a_3, a_4, \ldots, a_n, \ldots$ are the terms of the sequence. When the domain of the function consists of the first $n$ positive integers only, the sequence is a finite sequence.

Example: Write the first four terms of the sequence given by the function.

1. $a_n = 2n+1$

$a_1 = 2(1)+1 = 3$
$a_2 = 2(2)+1 = 5$
$a_3 = 2(3)+1 = 7$
$a_4 = 2(4)+1 = 9$

2. $a_n = \frac{2+(-1)^n}{n}$

$a_1 = \frac{2+(-1)^1}{1} = \frac{1}{1} = 1$ $\quad a_3 = \frac{2+(-1)^3}{3} = \frac{1}{3}$
$a_2 = \frac{2+(-1)^2}{2} = \frac{3}{2}$ $\quad a_4 = \frac{2+(-1)^4}{4} = \frac{3}{4}$

The $n$th term *must be given* to define a unique sequence.

$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{15}, \ldots a_n \ldots$ $\quad a_n = \frac{6}{(n+1)(n^2-n+6)}$

$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots a_n \ldots$ $\quad a_2 = \frac{1}{2^n}$

**Finding the *n*th term of a Sequence**

Look for an apparent pattern for the sequence, then write the expression for the $n$th term.

Example:
3. Write an expression for that apparent $n$th term $(a_n)$ of each sequence.

a) $1, 5, 9, 13, \ldots$

adding 4 $\quad \boxed{a_n = 4n-3}$

| n: | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| term: | 1 | 5 | 9 | 13 |

b) $2, -4, 6, -8, \ldots$

every other is neg

$(-1)^n \cdot (-2n)$
$(-1)^n \cdot (-1)(2n)$

$\boxed{a_n = (-1)^{n+1} \cdot 2n}$

| n: | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| term: | 2 | -4 | 6 | -8 |

**Recursive Sequence:** To define a sequence recursively, you need to be given one or more of the first few terms. All other terms of the sequence are then defined using previous terms.

Example:
4. Write the first five terms of the sequence defined recursively as $a_1 = 6$, $a_{k+1} = a_k + 1$, where $k \geq 2$..

$a_1 = 6$
$a_2 = 6+1 = 7$ $\quad a_2 = a_{k+1} = a_1 + 1 = 6+1 = 7$
$a_3 = 7+1 = 8$ $\quad a_3 = a_{2+1} = a_2 + 1 = 7+1 = 8$
$a_4 = 8+1 = 9$
$a_5 = 9+1 = 10$

next term after $a_k$

Sometimes it is convenient to begin subscripting a sequence with 0 instead of 1 so that the terms of the sequence become $a_0, a_1, a_2, a_3, a_4, \ldots$. When this is the case, the domain includes 0.

**Fibonacci Sequence:** $\overset{a_0}{1}, \overset{a_1}{1}, \overset{a_2}{2}, \overset{a_3}{3}, 5, 8, 13, 21, \ldots$

$a_0 = 1$ , $a_1 = 1$

$a_k = a_{k-2} + a_{k-1}$ where $k \geq 2$

$a_2 = a_{2-2} + a_{2-1}$
$a_0 + a_1$
$1 + 1 = 2$

$a_3 = a_{3-2} + a_{3-1}$
$= a_1 + a_2$
$= 1 + 2 = 3$

$a_4 = a_{4-2} + a_{4-1}$
$= a_2 + a_3$
$= 2 + 3 = 5$

Example:

5. Write the first five terms of the sequence defined recursively as
$a_0 = 1,\ a_1 = 3,\ a_k = a_{k-2} + a_{k-1}$, where $k \geq 2$.

$\boxed{1, 3, 4, 7, 11}$

$a_0 = 1$

$a_1 = 3$

$a_2 = a_{2-2} + a_{2-1}$
$= a_0 + a_1$
$= 1 + 3$
$= 4$

$a_3 = a_{3-2} + a_{3-1}$
$= a_1 + a_2$
$= 3 + 4$
$= 7$

**Summation Notation:** The sum of the first $n$ terms of a sequence is represented by

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + a_4 + \ldots + a_n$$

where $i$ is called the index of summation, $n$ is the upper limit of summation, and 1 is the lower limit of summation.

upper limit $\rightarrow n$

$\sum_{i=1}^{n}$ means to find the sum.

lower limit

$\Sigma$: shorthand for adding terms of a sequence

$n$ is positive

Example:

8. Find the sum $\sum_{i=1}^{4}(4i+1)$.

$= [4(1)+1] + [4(2)+1] + [4(3)+1] + [4(4)+1]$
$= 5 + 9 + 13 + 17 = \boxed{44}$

**Properties of Sums:**

1. $\sum_{i=1}^{n} c = cn$ $\qquad \sum_{i=1}^{7} 4 \qquad 4 \cdot 7 = 28$

2. $\sum_{i=1}^{n} c a_i = c \sum_{i=1}^{n} a_i$

3. $\sum_{i=1}^{n} a_i + b_i = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$

4. $\sum_{i=1}^{n} a_i - b_i = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i$

$\sum_{i=1}^{3} i^2 + i = \sum_{i=1}^{3} i^2 + \sum_{i=1}^{3} i$

Sequence : list of #
series : sum of #

**Series:** The sum of a ~~series~~ sequence.

Consider the infinite sequence $a_1, a_2, a_3, \ldots, a_i, \ldots$

1. The sum of the first $n$ terms of the sequence is called a finite series or the ***n*th partial sum** of the sequence and is denoted by

$$a_1 + a_2 + a_3 + \cdots + a_n = \sum_{i=1}^{n} a_i$$

2. The sum of all the terms of the infinite sequence is called an infinite series and is denoted by

$$a_1 + a_2 + a_3 + \ldots + a_i + \ldots = \sum_{i=1}^{\infty} a_i$$

Example:

9. For the series $\sum_{i=1}^{\infty} \frac{5}{10^i}$ find:

a) the fourth partial sum

$$\sum_{i=1}^{4} \frac{5}{10^i} = \frac{5}{10^1} + \frac{5}{10^2} + \frac{5}{10^3} + \frac{5}{10^4}$$
$$= .5 + .05 + .005 + .0005$$
$$= \boxed{.5555}$$

b) the sum

$$\sum_{i=1}^{\infty} \frac{5}{10^i} = \frac{5}{10^1} + \frac{5}{10^2} + \frac{5}{10^3} + \frac{5}{10^4} + \ldots$$
$$= .5555555$$
$$= \boxed{\frac{5}{9}}$$

10. An investor deposits \$1000 in an account that earns 3% interest compounded monthly. The balance in the account after $n$ months is given by

$$A_n = 1000(1 + \tfrac{0.03}{12})^n, \quad n = 0, 1, 2, \ldots$$

a) Write the first three terms of the sequence.
b) Find the balance in the account after four years by computing the 48[th] term of the sequence.

a) $A_0 = 1000\left(1 + \frac{0.03}{12}\right)^0 = \$1000$

$A_1 = 1000\left(1 + \frac{0.03}{12}\right)^1 = \$1002.50$

$A_2 = 1000\left(1 + \frac{0.03}{12}\right)^2 = \$1005.01$

b) $A_{48} = 1000\left(1 + \frac{0.03}{12}\right)^{48}$

$= \$1127.33$

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