The **Graph of an Inequality** is the collection of all solutions of the inequality. The graph of the equation will usually separate the plane into two or more regions. In each region, one of the following must be true.

1. *All* points in the region are solutions of the inequality.
2. *No* point in the region is a solution of the inequality.

You can determine whether the points in an entire region satisfy the inequality by testing *one* point in the region.

Sketching the Graph of an Inequality in Two Variables

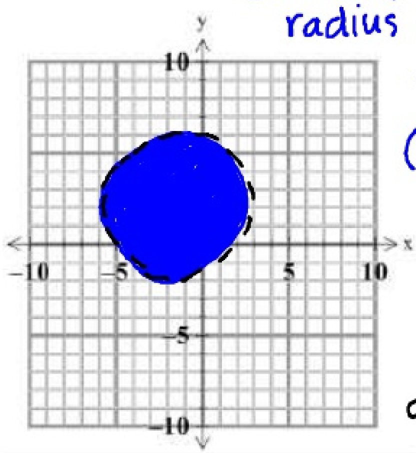
1. Replace the inequality sign by an equal sign and sketch the graph of the resulting equation. (Use a dashed line for $<, >$ and a solid line for $\le, \ge$.)

2. Test one point in each of the regions formed by the graph in Step 1. If the point satisfies the inequality, then shade the entire region to denote that every point in the region satisfies the inequality.

Examples:

1. Sketch the graph of $(x+2)^2+(y-2)^2<16$.

Center: $(-2,2)$
radius $=4$



Test $(0,0)$
$(2)^2+(-2)^2<16$?
$4+4<16$ ✓

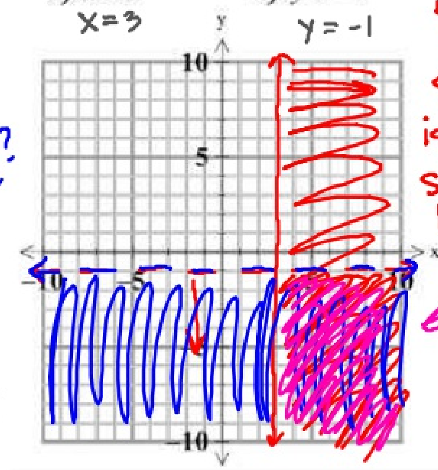
everything inside is the solution

dashed b/c $<$

2. Sketch the graph of each linear inequality.

a) $x \ge 3$ b) $y < -1$

$x=3$ $y=-1$



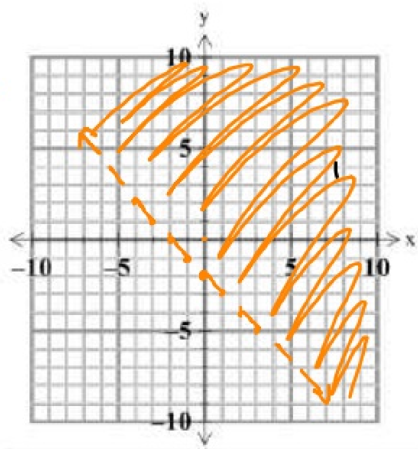
if a system then the solution is region shaded by both

3. Sketch the graph of $x+y>-2$.

$\quad -x \quad\; -x$

$y>-x-2$

dashed
$m=-1$
$(0,-2)$



$>$ shade above

Test $(0,0)$

$0>-(0)-2$?

$0>-2$ yes

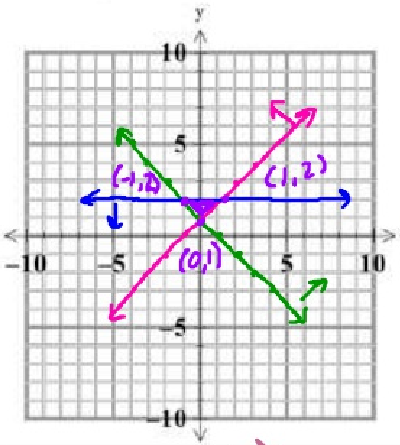
Systems of Inequalities

To sketch the graph of a system of inequalities in two variables, first sketch the graph of each individual inequality (on the same coordinate system) and then find the region that is __common__ to every graph in the system. This region represents the __Solution set__ of the system. For systems of *linear inequalities*, it is helpful to find the vertices of the solution region.

Examples:
Sketch the graph (and label the vertices) of the solution set of the system.

4. $\begin{cases} x+y\ge 1 \\ -x+y\ge 1 \\ y\le 2 \end{cases}$

$y \ge -x+1$ solid, above
$y \ge x+1$ solid, above
solid, below



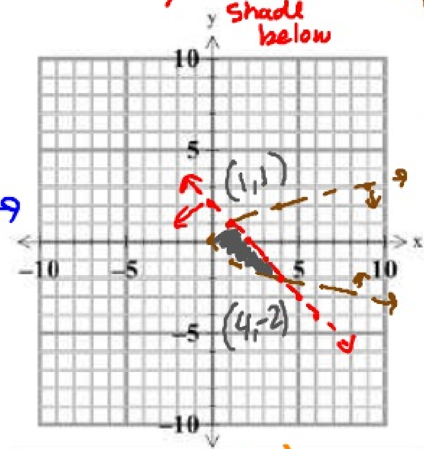
Bound

5. $\begin{cases} x-y^2>0 \\ x+y<2 \end{cases}$

$\rightarrow -y^2 = -x$
$y^2 = x$
$y = \pm\sqrt{x}$

$y<-x+2$ shade below

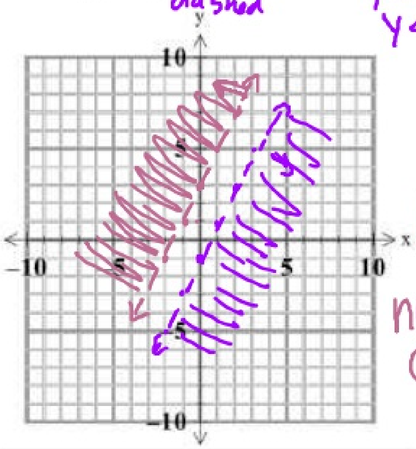
Test $(1,0)$
$1-(0)^2>0$?
yes
shade inside parabola



6. $\begin{cases} 2x-y<-3 \\ 2x-y>1 \end{cases}$

dashed
$-y<-2x-3$
$y>2x+3$

dashed
$-y>-2x+1$
$y<2x-1$



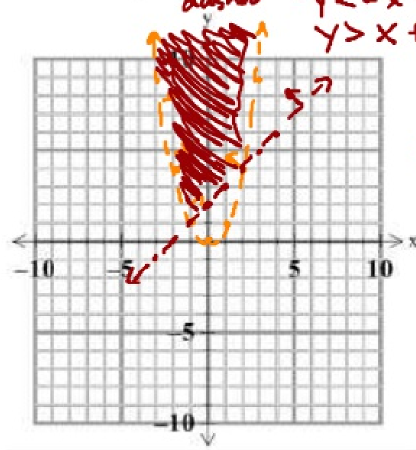
no overlap so $\boxed{\text{no solution}}$

no points in common

7. $\begin{cases} x^2-y<0 \\ x-y<-2 \end{cases}$

dashed
$-y<-x^2$
$y>x^2$

dashed
$-y<-x-2$
$y>x+2$



unbound

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