The Method of Elimination

1. *Obtain coefficients* for x (or y) that differ only in sign by multiplying all terms of one or both equations by suitably chosen constants.
2. *Add* the equations to eliminate one variable.
3. *Solve* the equation obtained in Step 2.
4. *Back-substitute* the value obtained in Step 3 into either of the original equations and solve for the other variable.
5. *Check* that the solution satisfies *each* of the original equations.

Examples:

1. Solve the system of equations using elimination.

$$+\begin{cases} 2x + \cancel{y} = 4 \\ 2x - \cancel{y} = -1 \end{cases}$$

$$\frac{4x}{4} = \frac{3}{4}$$

$$x = \frac{3}{4}$$

$$2\left(\frac{3}{4}\right) + y = 4$$

$$\frac{6}{4} + y = 4$$

$$-\frac{3}{2} \qquad -\frac{3}{2}$$

$$y = 2\tfrac{1}{2} \text{ or } \frac{5}{2}$$

$$\boxed{\left(\frac{3}{4}, \frac{5}{2}\right)}$$

2. $\begin{cases} 2x + 3y = 17 \\ (5x - y = 17)\cdot 3 \end{cases}$

$$15x - 3y = 51$$

$$+\ 2x + 3y = 17$$

$$\frac{17x}{17} = \frac{68}{17}$$

$$x = 4$$

$$2(4) + 3y = 17$$

$$-8 \qquad -8$$

$$\frac{3y}{3} = \frac{9}{3}$$

$$y = 3$$

$$\boxed{(4,3)}$$

3. $\begin{cases} (3x + 2y = 7)\cdot 2 \\ (2x + 5y = 1)\cdot(-3) \end{cases}$

$$6x + 4y = 14$$

$$-6x - 15y = -3$$

$$\frac{-11y}{-11} = \frac{11}{-11}$$

$$y = -1$$

$$3x + 2(-1) = 7$$

$$+2 \qquad +2$$

$$\frac{3x}{3} = \frac{9}{3}$$

$$x = 3$$

$$\boxed{(3,-1)}$$

4. $\begin{cases} (0.03x + 0.04y = 0.75)(200) \\ (0.02x + 0.06y = 0.90)(-300) \end{cases}$

$$6x + 8y = 150$$

$$-6x - 18y = -270$$

$$\frac{-10y}{-10} = \frac{-120}{-10}$$

$$y = 12$$

$$6x + 8(12) = 150$$

$$-96 \qquad -96$$

$$\frac{6x}{6} = \frac{54}{6}$$

$$x = 9$$

$$\boxed{(9,12)}$$

Graphical Approach to Finding a Solution

A system of linear equations is Consistent when it has at least one solution.

A consistent system with exactly one solution is independent, whereas a consistent system with infinitely many solutions is dependent.

A system is inconsistent when it has no solutions.

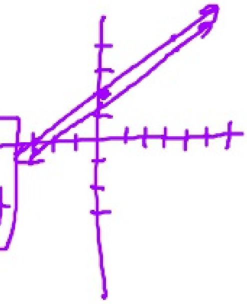
Examples. Sketch the graph of the system of linear equations. Then describe the number of solutions and state whether the system is consistent or inconsistent.

5. $\begin{cases} -2x+3y=6 \\ 4x-6y=-9 \end{cases}$ $\rightarrow$ $3y = 2x+6$

$y = \frac{2}{3}x + 2$

$\frac{-6y}{-6} = \frac{-4x}{-6} - \frac{9}{-6}$

$y = \frac{2}{3}x + \frac{3}{2}$



$\boxed{\text{no solution} \\ \text{inconsistent}}$

6. $\begin{cases} 6x-5y=3 \\ -12x+10y=5 \end{cases}$

$\frac{-5y}{-5} = \frac{-6x}{-5} + \frac{3}{-5}$

$y = \frac{6}{5}x - \frac{3}{5}$

$\frac{10y}{10} = \frac{12x}{10} + \frac{5}{10}$

$y = \frac{6}{5}x + \frac{1}{2}$

$\boxed{\text{no solution} \\ \text{inconsistent}}$

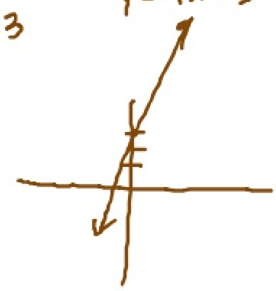
7. $\begin{cases} \left(\frac{1}{2}x - \frac{1}{8}y = -\frac{3}{8}\right) \cdot 8 \\ -4x+y=3 \end{cases}$

$4x - y = -3$

$-y = -4x - 3$

$y = 4x + 3$

$y = 4x+3$



$\boxed{\text{infinitely many solutions} \\ \text{consistent (dependent)}}$

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