A SYSTEM OF EQUATIONS is a set of two or more equations with the same variables.

$$\begin{cases} 2x+y=5 \\ 3x-2y=4 \end{cases}$$

A SOLUTION(S) to the system are variable values that satisfy ALL of the equations.
(In other words, the solution values must make every equation of the system true.)

check (2,1)
$2(2)+(1)=5?$ ✓
$3(2)-2(1)=4?$ ✓
(2,1) is a Solution

check (0,5)
$2(0)+5=5?$ ✓
$3(0)-2(5)=4?$ X
(0,5) not a solution

The Method of Substitution

1. *Solve* one of the equations for one variable in terms of the other.
2. *Substitute* the expression found in Step 1 into the other equation to obtain an equation in one variable.
3. *Solve* the equation obtained in Step 2.
4. *Back-substitute* the value obtained in Step 3 into the expression obtained in Step 1 to find the value of the other variable.
5. *Check* that the solution satisfies *each* of the original equations.

Examples:

1. Solve the system of equations using substitution.

$$\begin{cases} x-y=0 \\ 5x-3y=6 \end{cases}$$

$x=y$

$5(y)-3y=6$

$\frac{2y}{2}=\frac{6}{2}$

$y=3$

$x=3$

$\boxed{(3,3)}$

2. A total of $25,000 is invested in two funds paying 6.5% and 8.5% simple interest. The yearly interest is $2000. How much is invested at each rate?

x = $ invested at 6.5%
y = " " 8.5%

$x+y=25{,}000$

$1000\,(.065x+.085y=2{,}000)$

$65x+85y=2{,}000{,}000$

$x=-y+25{,}000$

$65(-y+25000)+85y=2{,}000{,}000$

$-65y+1{,}625{,}000+85y=2{,}000{,}000$

$\frac{20y}{20}=\frac{375{,}000}{20}$

$y=18{,}750$

$\boxed{\begin{array}{l}\$18{,}750 \text{ at } 8.5\% \\ \$6{,}250 \text{ at } 6.5\%\end{array}}$

3. $\begin{cases} -2x+y=5 \\ x^2-y+3x=1 \end{cases}$ $\longrightarrow y=2x+5$

$x^2-(2x+5)+3x=1$

$x^2-2x-5+3x=1$
$\quad -1 \qquad -1$

$x^2+x-6=0$ $\quad \frac{x}{-6} \quad \frac{+}{1}$

$(x+3)(x-2)=0$

$x=-3,2$

-3: $y=2(-3)+5$
$y=-6+5=-1$

2: $y=2(2)+5$
$=4+5=9$

$\boxed{\begin{matrix}(-3,-1)\\(2,9)\end{matrix}}$

4. $\begin{cases} 2x-y=-3 \\ 2x^2+4x-y^2=0 \end{cases}$

$-y=-2x-3$
$y=2x+3$

$2x^2+4x-(2x+3)^2=0$

$2x^2+4x-[(2x+3)(2x+3)]=0$

$2x^2+4x-(4x^2+12x+9)=0$

$2x^2+4x-4x^2-12x-9=0$

$-2x^2-8x-9=0$

$2x^2+8x+9=0$ $\quad \frac{x}{18} \quad \frac{+}{8}$

$x=\frac{-8\pm\sqrt{64-72}}{4}$

$x=\frac{-8\pm\sqrt{-8}}{4}$

$\boxed{\text{no real solutions}}$

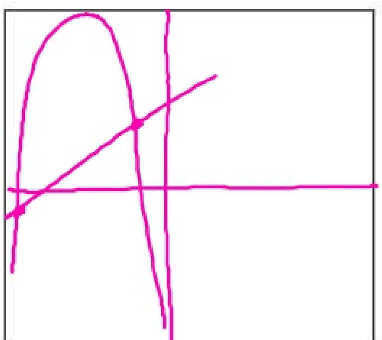
Graphical Approach to Finding a Solution

One method of finding solutions to systems is to graph the equations and find their intersections.

The intersection(s) is a solution because the equations have the same x- and y-values at that point.
(x, y)

Use a calculator to graph each system. Find a window size that shows ALL intersections, sketch a picture that shows ALL intersections, then determine the solution(s). *round to 2 decimal places*

1. $\begin{cases} y=x+7 \\ y=-x^2-10x-15 \end{cases}$



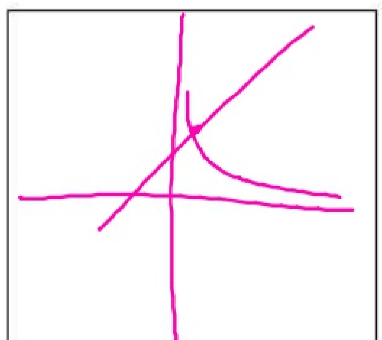
Window:

xmin: -10 xmax: 10

ymin: -10 ymax: 10

Solution(s): $(-2.63, 4.37), (-8.37, -1.37)$

2. $\begin{cases} y=3-\log x \\ -2x+y=1 \end{cases}$



Window:

xmin: -10 xmax: 10

ymin: -10 ymax: 10

Solution(s): $(1,3)$

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