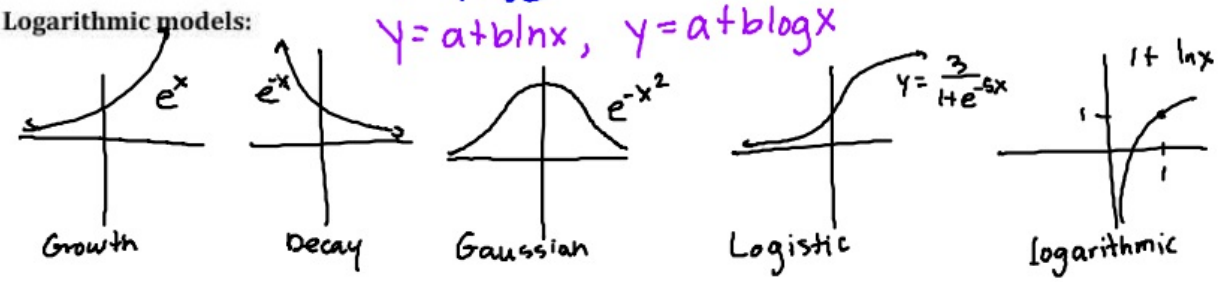
The five most common types of mathematical models involving exponential functions and logarithmic functions are as follows.

1. **Exponential growth model:** $y = ae^{bx}$ $b > 0$ positive exponent
2. **Exponential decay model:** $y = ae^{-bx}$ $b > 0$ negative exponent
3. **Gaussian model:** $y = ae^{-(x-b)^2/c}$
4. **Logistic growth model:** $y = \frac{a}{1+be^{-rx}}$
5. **Logarithmic models:** $y = a + b\ln x$, $y = a + b\log x$



Exponential Growth and Decay

Example:

1. In Example 1 (page 239), when will the amount of U.S. online advertising spending reach \$100 billion?

$t = 11 \Rightarrow 2011$

$\frac{100}{9.30} = \frac{9.30e^{0.1129t}}{9.30}$

$10.752688 = e^{0.1129t}$

$\ln(10.752688) = 0.1129t$

$2.375155786 = 0.1129t$

$t = \frac{2.375155786}{.1129}$

$t \approx 21$

2021

2. The number of bacteria in a culture is increasing according to the law of exponential growth. After 1 hour there are 100 bacteria, and after 2 hours there are 200 bacteria. How many bacteria will there be after 3 hours?

$y = ae^{bt}$

$y = 50e^{.693(3)}$

≈ 399.8

or about 400 bacteria

$100 = ae^{1b}$

$\frac{100}{e^b} = a$

$200 = ae^{2b}$

$200 = \frac{100}{e^b} \cdot e^{2b}$

$\frac{e^{2b}}{e^b} = e^b$

$\frac{200}{100} = \frac{100e^b}{100}$

$2 = e^b \Rightarrow \ln 2 = b$

$\log_e 2$

$b \approx .693$ rate of growth

$\frac{100}{e^{.693}} = a$

$a \approx 50$

started w/ 50

3. Estimate the age of a newly discovered fossil for which the ratio of carbon 14 to carbon 12 is $R = 1/10^{14}$.

Given: $R = \frac{1}{10^{12}} e^{-t/8223}$

$10^{12} \cdot \frac{1}{10^{14}} = 10^{12} \cdot \frac{1}{10^{12}} e^{-t/8223}$

$\frac{1}{10^2} = e^{-t/8223}$

$\ln \frac{1}{100} = -\frac{t}{8223}$ (-8223)

(-8223)

$t \approx 37868.31444$

about 38000 years

Gaussian Models

Example:

4. In 2011, the SAT critical reading scores for high school graduates in the United States roughly followed the normal distribution given by $y = 0.0035e^{-(x-497)^2/25{,}992}, 200 \le x \le 800$ where x is the SAT score for critical thinking. Sketch the graph of this function. From the graph, estimate the average SAT critical reading score.

Logistic Growth Models

Example:

5. In Example 5 (p. 243), after how many days are 250 students infected?

Logarithmic Models

6. Find the intensities of earthquakes whose magnitudes are (a) $R = 6.0$ and (b) $R = 79$.

p. 245: 7, 11, 17, 29, 31, 37, [redacted]