There are two basic strategies for solving exponential and logarithmic equations. The first is based on the One-to-One Properties. The second is based on the Inverse Properties. For $a > 0$ and $a \neq 1$, the following properties are true for all x and y for which $\log_a x$ and $\log_a y$ are defined.

$\log_a x = y$ iff $a^y = x$

One-to-One Properties

$a^x = a^y$ if and only if $x = y$

$\log_a x = \log_a y$ iff $x = y$

Inverse Properties

$a^{\log_a x} = x$

$\log_a a^x = x$

Solving Simple Equations

Example:

1. Solve each equation for x.

a. $2^x = 512$

$2^x = 2^9$

$\boxed{x=9}$

b. $\log_6 x = 3$

$6^3 = x$ $\boxed{x=216}$

$6^{\log_6 x} = 6^3$

$x = 6^3$

c. $5 - e^x = 0$

$5 = e^x$

$\boxed{\ln 5 = x}$

$\boxed{\approx 1.609}$

d. $9^x = \frac{1}{3}$

$(3^2)^x = 3^{-1}$

$3^{2x} = 3^{-1}$

$2x = -1$

$\boxed{x = -\frac{1}{2}}$

Strategies for Solving Exponential and Logarithmic Equations

1. Rewrite the original equation in a form that allows the use of the One-to-One Properties of exponential or logarithmic functions.
2. Rewrite an *exponential* equation in logarithmic form and apply the Inverse Property of logarithmic functions.
3. Rewrite a *logarithmic* equation in exponential form and apply the Inverse Property of exponential functions.

Solving Exponential Equations

Examples:

2. Solve each equation and approximate the result to three decimal places, if necessary.

a. $e^{2x} = e^{x^2-8}$

$2x = x^2 - 8$

$0 = x^2 - 2x - 8$ $\quad \frac{\times}{-8} \quad \frac{+}{-2}$

$0 = (x-4)(x+2)$

$\boxed{x = 4, -2}$

b. $\frac{2(5^x)}{2} = \frac{32}{2}$

$5^x = 16$

$\log_5 16 = x$

$x = \frac{\log 16}{\log 5} \approx \boxed{1.723}$

3. Solve $e^x - 7 = 23$ and approximate the result to three decimal places.

$+7 \quad +7$

$e^x = 30$

$\ln 30 = x$

$\log_e$

$x \approx \boxed{3.401}$

get the exponential piece alone before changing to a log

4. Solve $6(2^{t+5})+4=11$ and approximate the result to three decimal places.

$-4 \quad -4$

$\frac{\cancel{6}(2^{t+5})}{\cancel{6}}=\frac{7}{6}$

$2^{t+5}=\frac{7}{6}$

$\log_2\left(\frac{7}{6}\right)=t+5$

$-5 \quad -5$

$\log_2\left(\frac{7}{6}\right)-5=t$

$t\approx\boxed{-4.778}$

*change of base

$\frac{\log\left(\frac{7}{6}\right)}{\log 2}-5$

5. Solve $e^{2x}-7e^x+12=0$. Solve like a quadratic. Use "u" substitution.

$u=e^x$

$u^2-7u+12=0$ $\quad \frac{x}{12} \; \frac{+}{-7}$

$(u-3)(u-4)=0$

$(e^x-3)(e^x-4)=0$

$e^x-3=0$, $e^x=3$, $\boxed{\ln 3=x \approx 1.099}$

$e^x-4=0$, $e^x=4$, $\boxed{\ln 4=x \approx 1.386}$

example

X^4-5x^2+6

$u=x^2$

u^2-5u+6

Solving Logarithmic Equations
Examples:

6. Solve each equation.

a. $\ln x=\frac{2}{3}$

$\boxed{e^{\frac{2}{3}}=x}$

$\boxed{\approx 1.948}$

b. $\log_2(2x-3)=\log_2(x+4)$

$2x-3=x+4$

$-x+3 \quad -x+3$

$\boxed{x=7}$

c. $\log 4x-\log(12+x)=\log 2$

condense

$\log\left(\frac{4x}{12+x}\right)=\log 2$

$(12+x)\cdot\frac{4x}{12+x}=2(12+x)$

$4x=24+2x$

$-2x \quad -2x$

$2x=24$

$\boxed{x=12}$

7. Solve $7+3\ln x=5$ and approximate the result to three decimal places.

$-7 \quad -7$

get log piece alone

$\frac{3\ln x}{3}=-\frac{2}{3}$

$\ln x=-\frac{2}{3}$

$e^{-\frac{2}{3}}=x$

$\boxed{x\approx 0.513}$

8. Solve $3\log_4 6x=9$.

$\frac{3\log_4 6x}{3}=\frac{9}{3}$

$\log_4 6x=3$

$4^3=6x$

$\frac{64}{6}=\frac{6x}{6}$

$\boxed{\frac{32}{3}=x}$

$\boxed{x\approx 10.667}$

9. Solve $\log x+\log(x-9)=1$.

base is 10

$\log[x(x-9)]=1$

$10^1=x^2-9x$

$-10 \quad -10$

$0=x^2-9x-10$

$(x+1)(x-10)=0$

$x=\cancel{-1}, 10$

not in domain

$\boxed{x=10}$

10. You invest \$500 at an annual interest rate of 5.25%, compounded continuously. How long will it take your money to double?

P $\quad r=.0525 \quad A=1000$

$A=Pe^{rt}$

$\frac{1000}{500}=\frac{500e^{.0525t}}{500} \Rightarrow 2=e^{.0525t}$

$\frac{\ln 2}{.0525}=\frac{.0525t}{.0525}$

$t=\frac{\ln(2)}{.0525}$

$t\approx\boxed{13.2 \text{ years}}$

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