Change-of-Base: Let a, b, and x be positive real numbers such that $a \neq 1$ and $b \neq 1$. Then $\log_a x$ can be converted to a different base as follows.

Base b	Common Base 10	natural Base e
$\log_a x = \frac{\log_b x}{\log_b a}$	$\log_a x = \frac{\log x}{\log a}$	$\log_a x = \frac{\ln x}{\ln e}$

Examples:

1. Evaluate $\log_2 12$ using the change-of-base formula and common logarithms.

$$\frac{\log 12}{\log 2} \approx 3.585$$

2. Evaluate $\log_2 12$ using the change-of-base formula and natural logarithms.

$$\frac{\ln 12}{\ln 2} \approx 3.585$$

Properties of Logarithms

Let a be a positive number such that $a \neq 1$, and let n be a real number. If u and v are positive real numbers, then the following properties are true.

	Logarithm with Base a	Natural Logarithm
1. Product Property: $a^m \cdot a^n = a^{m+n}$	$\log_a(uv) = \log_a u + \log_a v$	$\ln(uv) = \ln u + \ln v$
2. Quotient Property: $\frac{a^m}{a^n} = a^{m-n}$	$\log_a\left(\frac{u}{v}\right) = \log_a u - \log_a v$	$\ln\left(\frac{u}{v}\right) = \ln u - \ln v$
3. Power Property: $(a^m)^n = a^{m \cdot n}$	$\log_a u^n = n \cdot \log_a u$	$\ln u^n = n \cdot \ln u$

Examples:

3. Write each logarithm in terms of $\log 3$ and $\log 5$.

a. $\log 75$

$\log(5^2 \cdot 3)$

$\log 5^2 + \log 3$

$2\log 5 + \log 3$

b. $\log \frac{9}{125}$ $= \log \frac{3^2}{5^3}$

$= \log 3^2 - \log 5^3$

$= 2\log 3 - 3\log 5$

4. Find the exact value of $\ln e^6 - \ln e^2$ without using a calculator.

$6 - 2 = 4$

Rewriting Logarithmic Expressions

The properties of logarithms are useful for rewriting logarithmic expressions in forms that simplify the operations of algebra.

Expanding Logarithmic Expressions

Examples:

5. Expand each logarithmic expression.

a. $\log_4(5x^3y)$

$= \log_4 5 + \log_4 x^3 + \log_4 y$

$= \boxed{\log_4 5 + 3\log_4 x + \log_4 y}$

b. $\log_3 \frac{4x^2}{\sqrt{y}}$

$y^{\frac{1}{2}}$

$\log_3(4x^2) - \log_3 \sqrt{y}$

$\log_3 4 + \log_3 x^2 - \log_3 y^{\frac{1}{2}}$

$\boxed{\log_3 4 + 2\log_3 x - \frac{1}{2}\log_3 y}$

Condensing Logaritmic Expressions

Examples:

a. $\frac{1}{2}\ln(x+2) - \ln x$

$\ln(x+2)^{\frac{1}{2}} - \ln x$

$\ln\sqrt{x+2} - \ln x$

$\boxed{\ln \frac{\sqrt{x+2}}{x}}$

b. $2[\log(x+3) - 2\log(x-2)]$

$2\log(x+3) - 4\log(x-2)$

$\log(x+3)^2 - \log(x-2)^4$

$\boxed{\log \frac{(x+3)^2}{(x-2)^4}}$

$\sqrt[n]{a} = a^{\frac{1}{n}}$

$\log\left(\frac{(x+3)}{(x-2)^2}\right)^2$

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