Definition of Logarithmic Function with base *a*:

$2 = \log_3 9$

$9 = 3^2$

For $x > 0$, $a > 0$, and $a \neq 1$,

$y = \log_a x$ if and only if $x = a^y$.

$a^0 = 1$

$f(x) = \log_a x$ "log base a of x"

Example:

1. Evaluate each logarithm at the indicated value of *x*.

a. $f(x) = \log_2 x$, $x = 1$

$f(1) = \log_2(1)$

$2^y = 1$

$y = \boxed{0}$

b. $f(x) = \log_5 x$, $x = \frac{1}{125}$

$f(\frac{1}{125}) = \log_5 \frac{1}{125}$

$5^y = \frac{1}{125}$

$y = \boxed{-3}$

c. $f(x) = \log_{10} x$, $x = 10,000$

$f(10,000) = \log_{10} 10,000$

$10^y = 10,000$

$\boxed{y = 4}$

Common Logarithm

$\log_{10} \rightarrow \log$

Example:

2. Use a calculator to evaluate the function $f(x) = \log x$. base 10

a. $x = 275$

$\log 275 \approx \boxed{2.439}$

b. $x = 0.275$

$\log 0.275 \approx \boxed{-0.561}$

c. $x = -\frac{1}{2}$

$\log(-\frac{1}{2}) =$ Error

d. $x = \frac{1}{2}$

$\log(\frac{1}{2}) = \boxed{-0.301}$

Properties of Logarithms

1. $\log_a 1 = 0$ because $a^0 = 1$
2. $\log_a a = 1$ because $a^1 = a$
3. $\log_a a^x = x \cdot \log_a a = x$ and $a^{\log_a x} = x$
4. If $\log_a x = \log_a y$, then $x = y$.

Examples:

3. Simplify each of the following:

a. $\log_9 9 = \boxed{1}$

#2

b. $20^{\log_{20} 3} = \boxed{3}$

#3

c. $\log_{\sqrt{5}} 1 = \boxed{0}$

#1

4. Solve $\log_5(x^2 + 3) = \log_5 12$ for x.

#4

$x^2 + 3 = 12$

$-3 \quad -3$

$x^2 = 9$

$x = \pm 3$

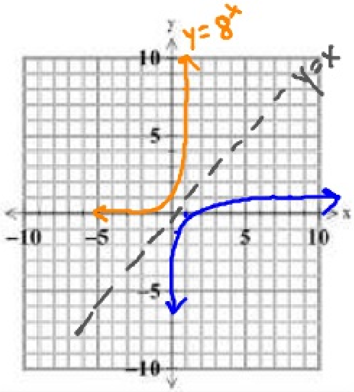
-3 is an extraneous solution

$\boxed{x = 3}$

Graphs of Logarithmic Functions

Examples:

5. In the same coordinate plane, sketch the graphs of (a) $f(x) = 8^x$ and (b) $g(x) = \log_8 x$.



$y = 8^x$

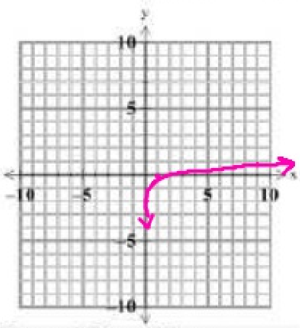
x	y
-1	$8^{-1} = \frac{1}{8}$
0	$8^0 = 1$
1	$8^1 = 8$

since a logarithm is the inverse of an exponential we can switch x & y values to get $\log_8 X$

$\log_8 X$

X	Y
$\frac{1}{8}$	-1
1	0
8	1

6. Sketch the graph of $f(x) = \log_9 x$. Identify the vertical asymptote.



V.A.: X=0

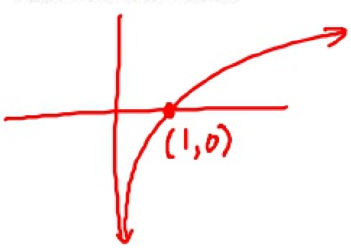
$y = 9^x$

X	Y
-1	$\frac{1}{9}$
0	1
1	9

$\log_9 X$

X	Y
$\frac{1}{9}$	-1
1	0
9	1

Parent Function:



Graph of $y = \log_a x$, $a > 1$

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

x-intercept: (1,0)

Increasing

V.A. X=0 (y-axis)

Continuous

inverse of exponential so reflection over y=x

Example

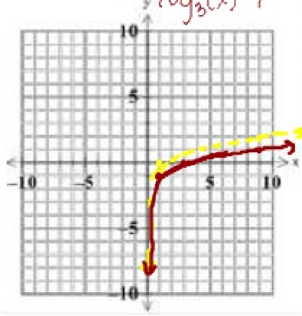
7. Use the graph of $f(x) = \log_3 x$ to sketch the graph of each function.

a. $g(x) = -1 + \log_3 x$ down 1

$= y \log_3(x) - 1$

X	Y
1	0
3	1
9	2

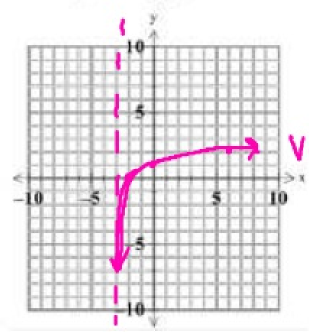
X	Y
1	-1
3	0
9	1



V.A. X=0

b. $h(x) = \log_3(x+3)$ left 3

X	Y
-2	0
0	1
6	2



V.A. X=-3

The Natural Logarithmic Function

$\log_e \to \ln$ $\qquad$ $\log_e x \to \ln x$ $\qquad$ $x>0$

Example:

8. Use a calculator to evaluate the function $f(x)=\ln x$ at each value of x.

a. $x=0.01$ $\qquad$ b. $x=4$ $\qquad$ c. $x=\sqrt{3}+2$ $\qquad$ d. $x=\sqrt{3}-2$

$\ln(0.01)\approx\boxed{-4.605}$ $\quad$ $\ln(4)\approx\boxed{1.386}$ $\quad$ $\ln(\sqrt{3}+2)\approx\boxed{1.317}$ $\quad$ $\ln(\sqrt{3}-2)$ Not possible

Properties of Natural Logarithms

1. $\ln 1 = 0$ because $e^0=1$
2. $\ln e = 1$ because $e^1=e$
3. $\underline{\ln e}^x = x\cdot\ln e = \underline{x}$ and $e^{\ln x}=x$
4. If $\ln x=\ln y$, then $x=y$.

Examples:

9. Use the properties of natural logarithms to simplify each expression.

a. $\ln e^{\frac{1}{3}}$ #3 $\qquad$ b. $5\ln 1$ #1 $\qquad$ c. $\frac{3}{4}\ln e$ #2 $\qquad$ d. $e^{\ln 7}$ #3

$=\boxed{\frac{1}{3}}$ $\qquad$ $=5\cdot 0$, $=\boxed{0}$ $\qquad$ $=\frac{3}{4}\cdot 1$, $=\boxed{\frac{3}{4}}$ $\qquad$ $=\boxed{7}$

$x^2>4$

$x^2-4>0$

$(x+2)(x-2)>0$

$+ \quad -2 \quad - \quad 2 \quad +$

$(-\infty,-2)\cup(2,\infty)$

$x^2>0$

$x>0$ or $x<0$

$(-\infty,0)\cup(0,\infty)$

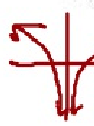
10. Find the domain of each function. $\qquad$ $\log_a x$ $\quad$ $x>0$

a. $f(x)=\ln(x-2)$ $\qquad$ b. $g(x)=\ln(2-x)$ $\qquad$ c. $h(x)=\ln x^2$ $\qquad$ d. $f(x)=\ln(x+3)$

a. $x-2>0$, $x>2$, $\boxed{(2,\infty)}$

b. $2-x>0$, $2>x$, $x<2$, $\boxed{(-\infty,2)}$

c. $\boxed{(-\infty,0)\cup(0,\infty)}$



$x\neq 0$

d. $x+3>0$, $x>-3$, $\boxed{(-3,\infty)}$

11. In Example 11, find the average score at the end of

(a) $t = 1$ month $\qquad$ (b) $t = 9$ months $\qquad$ (c) $t = 12$ months.

$f(t)=75-6\ln(t+1)$

(a) $f(1)=75-6\ln(1+1)$

$=75-6\ln(2)$

$\approx\boxed{70.84}$

(b) $f(9)=75-6\ln(9+1)$

$\approx\boxed{61.18}$

(c) $f(12)=75-6\ln(12+1)$

$\approx\boxed{59.61}$

p. 218: 7-20, 23, 25-28, 29, 32, 34, 35, 37-40, 41, 44, 45, 51, 55, 58, 60, 61, 62, 65, 67, 69, 72, 73, 75