Definition of Exponential Function: The **exponential function f with base a** is denoted by $f(x) = a^x$ where $a > 0,\ a \neq 1$, and x is any real number.

$y = 2^x \qquad f(x) = 3(4)^{x-1} + 2$

Examples:

1. Use a calculator to evaluate $f(x) = 8^{-x}$ at $x = \sqrt{2}$.

$f(\sqrt{2}) = 8^{-\sqrt{2}} = 0.053$

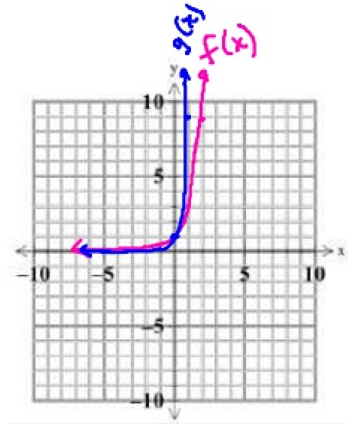
2. In the same coordinate plane, sketch the graph of each function.

a. $f(x) = 3^x$

x	y
-2	$3^{-2} = \frac{1}{9}$
-1	$3^{-1} = \frac{1}{3}$
0	$3^0 = 1$
1	$3^1 = 3$
2	$3^2 = 9$

b. $g(x) = 9^x$

x	y
-2	$9^{-2} = \frac{1}{81}$
-1	$9^{-1} = \frac{1}{9}$
0	$9^0 = 1$
1	$9^1 = 9$
2	$9^2 = 81$



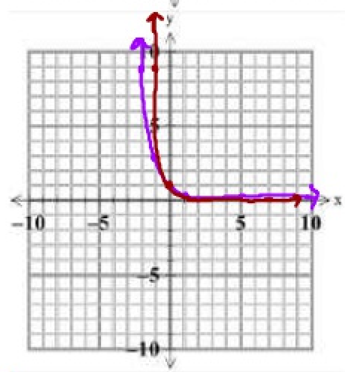
3. In the same coordinate plane, sketch the graph of each function.

a. $F(x) = 3^{-x}$ $\left(\frac{1}{3}\right)^x$

x	y
-2	$3^{-(-2)} = 3^2 = 9$
-1	$3^{-(-1)} = 3^1 = 3$
0	$3^0 = 1$
1	$3^{-1} = \frac{1}{3}$
2	$3^{-2} = \frac{1}{9}$

b. $G(x) = 9^{-x}$ $\left(\frac{1}{9}\right)^x$

x	y
-2	81
-1	9
0	1
1	$\frac{1}{9}$
2	$\frac{1}{81}$



Parent Functions:

$y = a^x$ if $a > 1$

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

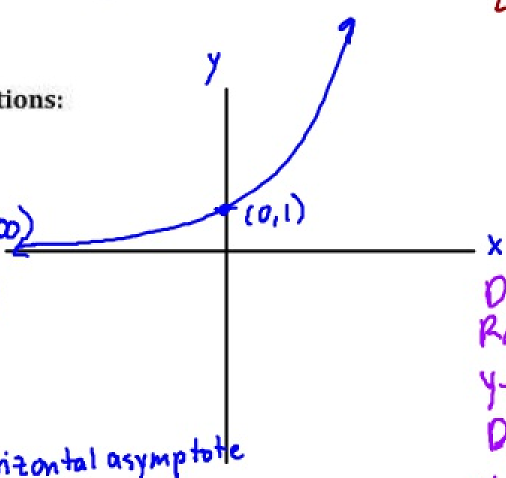
y-int: $(0,1)$

Increasing

x-axis is a horizontal asymptote

as $x \to -\infty$, $a^x \to 0$

Continuous



$y = a^x$ if $0 < a < 1$

$y = a^{-x}$ if $a > 1$

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

y-int: $(0,1)$

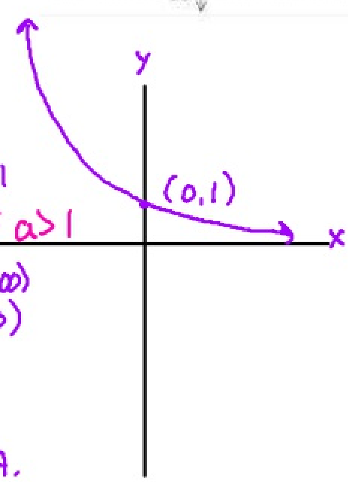
Decreasing

x-axis is a H.A.

as $x \to \infty$, $a^x \to 0$

$a^{-x} \to 0$

Continuous



One-to-One Property for $a>0$, $a \neq 1$

$a^x = a^y$ if and only if $x = y$

Examples:

4. Use the One-to-One Property to solve the equation for x.

Get same bases

a. $8 = 2^{2x-1}$

$2^3 = 2^{2x-1}$

$3 = 2x - 1$

$+1 \quad +1$

$4 = 2x$

$\boxed{x = 2}$

b. $\left(\frac{1}{3}\right)^{-x} = 27$

$(3^{-1})^{-x}$

$3^x = 3^3$

$\boxed{x = 3}$

5. Use the graph of $f(x) = 4^x$ to describe the transformation that yields the graph of each function.

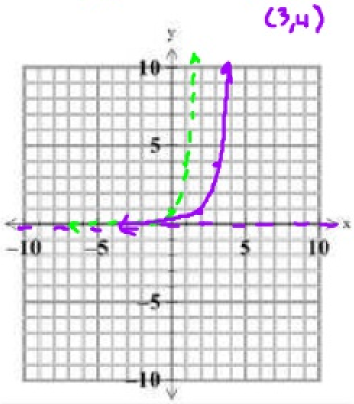
a. $g(x) = 4^{x-2}$

right 2

HA $y=0$

(2,1)

(3,4)



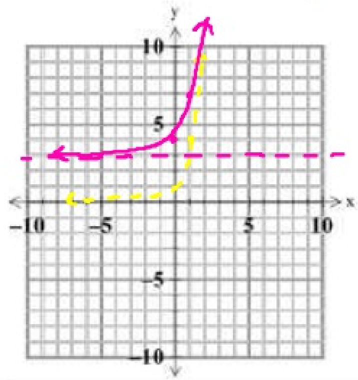
b. $h(x) = 4^x + 3$

up 3

HA: $y=3$

(0,4)

(1,7)



c. $k(x) = 4^{-x} - 3$

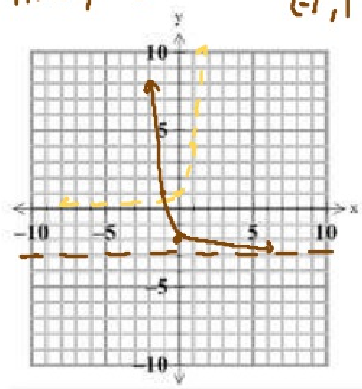
reflect over y-axis

down 3

H.A. $y=-3$

(0,-2)

(-1,1)



The Natural Base e: $e \approx 2.718281828\ldots$

Examples:

6. Use a calculator to evaluate the function $f(x) = e^x$ at each value of x.

a. $x = 0.3$

$f(0.3) = e^{0.3} = \boxed{1.350}$

b. $x = -1.2$

$f(-1.2) = e^{-1.2} = \boxed{0.301}$

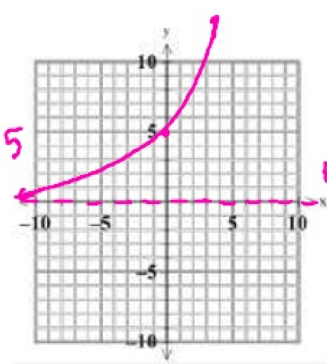
c. $x = 6.2$

$f(6.2) = e^{6.2} = \boxed{492.749}$

7. Sketch the graph of $f(x) = 5e^{0.17x}$.

vert. stretch by factor of 5

horiz. stretch/shrink



(0,5)

H.A. $y=0$

Formulas for Compound Interest

After t years, the balance A in an account with principal P and annual interest rate r (in decimal form) is given by the following formulas.

1. For n compoundings per year: $A = P\left(1+\frac{r}{n}\right)^{nt}$

2. For continuous compounding: $A = Pe^{rt}$

Examples : $P = 6000$ $r = .04$ $t = 7$

8. You invest \$6000 at an annual rate of 4%. Find the balance after 7 years when the interest is compounded

a. quarterly. $n=4$

$A = 6000\left(1+\frac{.04}{4}\right)^{(4)(7)}$

$A = \boxed{\$7927.75}$

b. monthly. $n=12$

$A = 6000\left(1+\frac{.04}{12}\right)^{(12)(7)}$

$A = \boxed{\$7935.08}$

c. continuously.

$A = 6000e^{(.04)(7)}$

$A = \boxed{\$7938.78}$

9. In Example 9, how much of the 10 pounds will remain in the year 2089? How much of the 10 pounds will remain after 125,000 years?

$1986 \rightarrow t = 0$

$P = 10\left(\frac{1}{2}\right)^{\frac{t}{24,100}}$

$2089 \rightarrow t = 103$

$P = 10\left(\frac{1}{2}\right)^{\frac{103}{24,100}}$

$P = 9.97$ lb

$P = 10\left(\frac{1}{2}\right)^{\frac{125,000}{24,100}}$

$P = 0.27$ lb

p. 208: 8, 11, 13-16, 22, 23-26, 28, 30, 31-33, 36, 37, 40, 41, 45, 49, 51, 54, 55, 58, 59, 61, 64, 67