A polynomial can change signs only at its zeros. Between two consecutive zeros, a polynomial must be entirely positive or entirely negative. The zeros are called Key numbers or Critical values.

Factor the polynomial and find the critical values. $x^2 - 2x - 3$

(Key)

$(x-3)(x+1)=0$

$\boxed{x = 3, -1}$

To solve the inequality $x^2 - 2x - 3 < 0$ (negative), test one value from each of the test intervals. When a value satisfies the original inequality, you can conclude that the interval is a solution of the inequality.

test -2: $(-2)^2 - 2(-2) - 3$, $4 + 4 - 3$ (+)

test 0: $(0)^2 - 2(0) - 3$ (−)

test 4: $(4)^2 - 2(4) - 3$, $16 - 8 - 3$ (+)

zeros
Key
critical

$\boxed{(-1, 3)}$

$\{x \mid -1 < x < 3\}$

"x is such that"

>0 positive

<0 negative

Examples: Solve the inequality, then graph the solution.

1. $x^2 - x - 6 < 0$

$(x-3)(x+2) < 0$

Key numbers: 3, -2

Test -3: $(-)(-) = +$, $(-3-3)(-3+2) = 6$

Test 0: $(-)(+) = -$

Test 4: $(+)(+) = +$

$\boxed{(-2, 3)}$

$\{x \mid -2 < x < 3\}$

2. $3x^3 - x^2 - 12x > -4$

$+4 \quad +4$

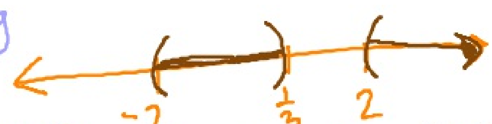
$3x^3 - x^2 - 12x + 4 > 0$ Factor by grouping

$x^2(3x-1) - 4(3x-1) > 0$ >0 means positive

$(3x-1)(x^2-4) > 0$

$(3x-1)(x+2)(x-2) > 0$

Key #s: $\frac{1}{3}, -2, 2$



Test -3: $(-)(-)(-) = -$

Test 0: $(-)(+)(-) = +$

Test 1: $(+)(+)(-) = -$

Test 3: $(+)(+)(+) = +$

$\boxed{(-2, \frac{1}{3}) \cup (2, \infty)}$

$\{x \mid -2 < x < \frac{1}{3} \text{ or } x > 2\}$

3. Solve $2x^2 + 3x < 5$ (a) algebraically and (b) graphically.

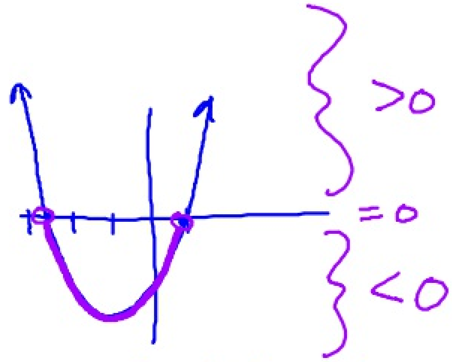
$2x^2 + 3x - 5 < 0$

$(2x+5)(x-1) < 0$

Key #s: $-\frac{5}{2}, 1$ (zeros)

Test -3	Test 0	Test 2
$(-)(-) = +$	$(+)(-) = -$	$(+)(+) = +$

$\boxed{\left(-\frac{5}{2}, 1\right)}$



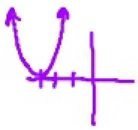
For what x-values are the y-values negative? <0

$\boxed{\left(-\frac{5}{2}, 1\right)}$

4. Unusual solutions sets.

(a) $x^2 + 6x + 9 < 0$

$(x+3)^2 < 0$



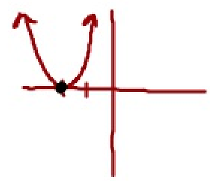
Key #: -3

Test -4	Test 0
$(-)^2 = +$	$(+)^2 = +$

$\boxed{\text{no solution}}$

(b) $x^2 + 4x + 4 \le 0$

$(x+2)^2 \le 0$



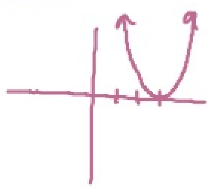
Test -3	Test 0
$(-)^2 = +$	$(+)^2 = +$

expression $= 0$ at -2

$\boxed{x = -2}$

(c) $x^2 - 6x + 9 > 0$

$(x-3)^2 > 0$



$\boxed{(-\infty, 3) \cup (3, \infty)}$

(d) $x^2 - 2x + 1 \ge 0$

$(x-1)^2 \ge 0$

Always true

$\boxed{(-\infty, \infty)}$

5. Rational Inequalities

(a) $\dfrac{x-2}{x-3} \ge -3$

Key #: $3, \frac{11}{4}$

$\dfrac{x-2}{x-3} + 3 \ge 0$

$\dfrac{x-2}{x-3} + 3\dfrac{(x-3)}{(x-3)} \ge 0$

$\dfrac{x-2}{x-3} + \dfrac{3x-9}{x-3} \ge 0$

$\dfrac{4x-11}{x-3} \ge 0$

$x \ne 3$

Test: 0
$- \div - = +$ ✓

Test: 2.9
$+ \div - = -$ X

Test: 4
$+ \div + = +$ ✓

$\boxed{\left(-\infty, \frac{11}{4}\right] \cup (3, \infty)}$

(b)

$\dfrac{5}{x-6} > \dfrac{3}{x+2}$

LCD: $(x-6)(x+2)$

$\dfrac{5}{x-6} - \dfrac{3}{x+2} > 0$

$\dfrac{5}{(x-6)} \cdot \dfrac{(x+2)}{(x+2)} - \dfrac{3}{(x+2)} \cdot \dfrac{(x-6)}{(x-6)} > 0$

Key #s: $6, -2, -14$

$\dfrac{5x+10-3x+18}{(x-6)(x+2)} > 0$

$\dfrac{2x+28}{(x-6)(x+2)} > 0$

$x \ne 6, -2$

$\boxed{(-14, -2) \cup (6, \infty)}$

Test: -15
$- \div + = -$ X

Test: -3
$+ \div + = +$ ✓

Test: 0
$+ \div - = -$ X

Test: 7
$+ \div + = +$ ✓

p. 187: 13, 17, 23, 27, 35, 37, 41, 43, 45, 49