Imaginary Unit *i*: $\sqrt{-1} = i$

$i^2 = -1$

Complex numbers – standard form:

Complex number

$a + bi$

real imaginary

$(b \neq 0)$ pure imaginary #

Real

imaginary

Complex

Equality of Complex Numbers: If $a+bi = c+di$ then $a=c$ and $b=d$

Operations with Complex Numbers:

Sum: $(a+bi)+(c+di) = (a+c)+(b+d)i$

Difference: $(a+bi)-(c+di) = (a-c)+(b-d)i$

Examples:

1. $(7+3i)+(5-4i)$

$= (7+5)+(3-4)i$

$= \boxed{12-i}$

2. $(3+4i)-(5-4i)$

$= (3-5)+(4+{}^{+}4)i$

$= \boxed{-2+8i}$

$(3+4i)-(5+4i)$

$\boxed{-2}$

3. $2i+(-3-4i)-(-3-3i)$

$= 2i + -3 - 4i + 3 + 3i$

$= \boxed{i}$

4. $(5-3i)+(3+5i)-(8+2i)$

$= 5-3i+3+5i-8-2i$

$= \boxed{0}$

5. $(2-4i)(3+3i)$

$= 6+6i-12i-12i^2$

$= 6-6i-12(-1)$

$= \boxed{18-6i}$

6. $(4+5i)(4-5i)$

$= 16-20i+20i-25i^2$

$= 16 + 0 \quad +25$

$= \boxed{41}$

7. $(4+2i)^2$

$= (4+2i)(4+2i)$

$= 16+16i+4i^2$

-4

$= \boxed{12+16i}$

Complex Conjugates: $(a+bi)(a-bi) = a^2 - abi + abi - b^2i^2$

$= a^2 - b^2(-1)$

$= \underline{a^2+b^2}$

sum of squares can only factor w/ imaginary #s

Examples: multiply by the conjugate

8. $3+6i$

$(3+6i)(3-6i)$

$= 9 - 18i + 18i - 36i^2$

$= 9+36$

$= \boxed{45}$

9. $2-5i$

$= (2-5i)(2+5i)$

$= (2)^2+(5)^2$

$= 4+25$

$= \boxed{29}$

10. Quotient → standard form ($a+bi$)

$$\frac{2+i}{2-i} \cdot \frac{(2+i)}{(2+i)} = \frac{4+4i+\overset{-1}{i^2}}{\underset{4}{2^2}+\underset{1}{1^2}} = \frac{3+4i}{5} = \boxed{\frac{3}{5}+\frac{4}{5}i}$$

↑ $2+i$ is the conjugate

Complex Solutions of Quadratic Equations: $\sqrt{-a} = \sqrt{a}\,i$ or $i\sqrt{a}$

$\sqrt{-1}\cdot\sqrt{a}$

Principal Square Root of a Negative Number – $i\sqrt{a}$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Examples:

11. Write $\sqrt{-14}\cdot\sqrt{-2}$ in st. form

$= i\sqrt{14}\cdot i\sqrt{2}$

$= i^2\sqrt{28}$ $<^4_7$

$\sqrt{28}$

$\sqrt{2\cdot 2 \cdot 7}$

$= 2(-1)\sqrt{7}$

$= \boxed{-2\sqrt{7}}$

$i = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$i^{31} = -i$ (i^{28}: 1, i^3: $-i$)

12. Solve.

$8x^2+14x+9=0$

$$x = \frac{-14 \pm \sqrt{(14)^2-4(8)(9)}}{2(8)}$$

$$x = \frac{-14 \pm \sqrt{196-288}}{16}$$

$$x = \frac{-14 \pm \sqrt{-92}}{16}$$

$$x = \frac{-14 \pm i\sqrt{92}}{16}$$

$$x = \frac{-14 \pm 2i\sqrt{23}}{16}$$

$$\frac{-14}{16} \pm \frac{2i\sqrt{23}}{16}$$

$$x = \frac{-7}{8} \pm \frac{i\sqrt{23}}{8}$$ OR $\boxed{\frac{-7}{8}+\frac{\sqrt{23}}{8}i \quad \frac{-7}{8}-\frac{\sqrt{23}}{8}i}$

2	92
2	46
23	23
	1

The Fundamental Theorem of Algebra:

If $f(x)$ is a polynomial of degree n, where $n>0$, then f has at least one zero in the complex number system.

Linear Factorization Theorem:

A polynomial of degree n $(n>0)$ will have n factors.

Example:

1. Determine the number of zeros in $f(x)=x^4-1$. $\boxed{4}$

The Rational Zero Test: $q x^n + \_x^{n-1} + \dots + \text{constant}$ (p is the constant)

possible rational zeros $\pm\frac{p}{q}$ $\qquad \pm\frac{\text{factors of constant}}{\text{factors of leading coefficient}}$

Examples: Find the rational zeros.

2. $f(x)=x^3-5x^2+2x+8$

$\pm\frac{\{1,2,4,8\}}{\{1\}}$ possible: $\pm1, \pm2, \pm4, \pm8$

$\boxed{x=-1, 2, 4}$

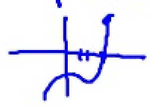
3. $f(x)=x^3+2x^2+6x-4$

possible: $\pm1, \pm2, \pm4$

$\boxed{\text{no rational roots}}$

4. $f(x)=x^3-3x^2+2x-6$

possible: $\pm1, \pm2, \pm3, \pm6$



$\boxed{x=3}$

5.

6. $f(x)=2x^4-9x^3-18x^2+71x-30$

$\frac{30:\ 1,2,3,5,6,10,15,30}{2:\ 1,2}$

possible: $\pm1, \pm2, \pm3, \pm5, \pm6, \pm10, \pm15, \pm30, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}$

$\boxed{x=-3, \tfrac{1}{2}, 2, 5}$

7. Find all real solutions

$x^3+4x^2-15x-18=0$

$x=-6, -1, 3$

Conjugate Pairs: $(a+bi)$ $(a-bi)$ $\qquad 2+\sqrt{3}$ $\quad 2-\sqrt{3}$

complex roots (zeros) always come in conjugate pairs

Finding a Polynomial Function with Given Zeros:

given $2i$ is a zero
then $-2i$ is also a zero

Examples: write the function given the zeros. (4th Degree)

8. $2, -2$, and $7i$ — $-7i$ is also a zero

$f(x) = (x-2)(x+2)(x-7i)(x+7i)$

$= (x^2-4)(x^2+49)$

$= x^4 + 49x^2 - 4x^2 - 196$

$\boxed{f(x) = x^4 + 45x^2 - 196}$

9. $1, 3$, and $4-i$ — $4+i$

$f(x) = (x-1)(x-3)(x-(4-i))(x-(4+i))$

$= (x-1)(x-3)(x-4+i)(x-4-i)$

$= (x^2-4x+3)(x^2-8x+17)$

$\boxed{f(x) = x^4 - 12x^3 + 52x^2 - 92x + 51}$

	x	-4	i
x	x^2	$-4x$	$\cancel{xi}$
-4	$-4x$	16	$\cancel{-4i}$
$-i$	$\cancel{-xi}$	$\cancel{4i}$	$-i^2$ = 1

	x^2	$-4x$	3
x^2	x^4	$-4x^3$	$3x^2$
$-8x$	$-8x^3$	$32x^2$	$-24x$
17	$17x^2$	$-68x$	51

[redacted].

Factoring a Polynomial:

Examples:

11. $g(x) = x^3 - 3x^2 + 7x - 5$

use a calc to find rational real root

$x = 1$

$$\begin{array}{r|rrrr} 1 & 1 & -3 & 7 & -5 \\ & & 1 & -2 & 5 \\ \hline & 1 & -2 & 5 & 0 \end{array}$$

$x^2 - 2x + 5$

$x = \frac{2 \pm \sqrt{4-20}}{2}$

$x = \frac{2 \pm \sqrt{-16}}{2}$

$x = \frac{2 \pm 4i}{2}$

$x = 1 \pm 2i$

zeros: $1, 1+2i, 1-2i$

factors: $(x-1)(x-1+2i)(x-1-2i)$

p. 152: 11, 15, 19, [redacted], 25, 27, 29, 33, [redacted], 39, [redacted], 43, 47, 51, [redacted], 55, 65, 71, [redacted], 75
p. 164: 9, 11, 12, 15, 18, 19, [redacted], 25, [redacted], 31, 37, [redacted], 41, 45, 47, 55, [redacted]