REVIEW: Simplify using long division. PAY ATTENTION TO THE PROCESS!

1. $2538 \div 6$

$$\begin{array}{r} \boxed{423} \\ 6\,\overline{)\,2538} \\ -24\downarrow \\ 13 \\ -12\downarrow \\ 18 \\ -18 \\ 0 \end{array}$$

2. $65413 \div 15$

$$\begin{array}{r} \boxed{4360\tfrac{13}{15}} \\ 15\,\overline{)\,65413} \\ -60\downarrow \\ 54 \\ -45\downarrow \\ 91 \\ -90\downarrow \\ 13 \\ -0 \\ 13 \end{array}$$

A **FACTOR** is a number or expression that evenly divides another number or expression. A number or expression evenly divides another if the remainder is zero.

Polynomials can be divided in the same manner as the numbers above.
REMEMBER TO ACCOUNT FOR ALL TERMS.

Simplify using long division and state whether the binomial is a factor of the polynomial.

3. $(3x^2+19x+28)\div(x+4)$

$$\begin{array}{r} \boxed{3x+7} \\ x+4\,\overline{)\,3x^2+19x+28} \\ -(3x^2+12x)\quad\downarrow \\ 7x+28 \\ -(7x+28) \\ 0 \end{array}$$

$(x+4)$ is a factor

$f(x)=(x+4)(3x+7)$

$\frac{3x^2+19x+28}{x+4}=3x+7$

4. $(6n^2+5n-10)\div(2n+7)$

$$\begin{array}{r} 3n-8+\frac{46}{2n+7} \\ 2n+7\,\overline{)\,6n^2+5n-10} \\ -(6n^2+21n)\quad\downarrow \\ -16n-10 \\ -(-16n-56) \\ 46 \end{array}$$

$f(x)=(2n+7)(3n-8)+46$

$\frac{6n^2+5n-10}{2n+7}=3n-8+\frac{46}{2n+7}$

Division Algorithm

$f(x)=d(x)q(x)+r(x)$

dividend divisor quotient remainder

$$\frac{f(x)}{d(x)}=q(x)+\frac{r(x)}{d(x)}$$

improper — higher power in numerator

proper — higher power in denominator

Long Division of Polynomials.

5. $(x^3-2x^2-9)\div(x-3)$

$x \neq 3$

$$\begin{array}{r} \boxed{x^2+x+3} \\ x-3\,\overline{)\,x^3-2x^2+0x-9} \\ -(x^3-3x^2)\quad\downarrow \\ x^2+0x \\ -(x^2-3x) \\ 3x-9 \\ -(3x-9) \\ 0 \end{array}$$

6. $(-x^3+9x+6x^4-x^2-3)\div(1+3x)$

$$\begin{array}{r} 2x^3-x^2+3+\frac{-6}{3x+1} \\ 3x+1\,\overline{)\,6x^4-x^3-x^2+9x-3} \\ -(6x^4+2x^3)\downarrow \\ -3x^3-x^2 \\ -(-3x^3-x^2) \\ 0+9x-3 \\ -(9x+3) \\ -6 \end{array}$$

SYNTHETIC DIVISION is a shorthand way of dividing a polynomial by a linear binomial $(x \pm c)$ by using only the Coefficients.

In order to use synthetic division, the dividend must be a polynomial written in standard form, ordered by the power of the variable, with the largest power listed first. If a term is missing, 0 must be used in its place.

7. $(5x^3 + 8x^2 - x + 6) \div (x+2)$

-2	5	8	-1	6
		-10	4	-6
	5_{x^2}	-2_x	3_c	0_r

$\boxed{5x^2 - 2x + 3}$

8. $\left(2n^4 - 5n^3 - 28n^2 + 24n - 45\right) \div (n-5)$

5	2	-5	-28	24	-45
		10	25	-15	45
	2	5	-3	9	0

$\boxed{2n^3 + 5n^2 - 3n + 9}$

We use division of polynomials to help factor and find all of the zeros.

The Remainder Theorem

If a polynomial $f(x)$ is divided by $x-k$, then the remainder is $r = f(k)$.

9. Use the Remainder Theorem to find each function value given.

$f(x) = 4x^3 + 10x^2 - 3x - 8$ $\div (x+1)$

a. $f(-1)$ $= 1$

-1	4	10	-3	-8
		-4	-6	9
	4	6	-9	1

$f(-1) = 4(-1)^3 + 10(-1)^2 - 3(-1) - 8$
$= -4 + 10 + 3 - 8$
$= 1$ ✓

b. $f(4)$ $= \boxed{396}$ $\div (x-4)$

$f(4) = 4(4)^3 + 10(4)^2 - 3(4) - 8$

4	4	10	-3	-8
		16	104	404
	4	26	101	396

$(x - \frac{1}{2})$

c. $\boxed{f\left(\frac{1}{2}\right) = -6.5}$

$\frac{1}{2}$	4	10	-3	-8
		2	6	1.5
	4	12	3	-6.5

d. $\boxed{f(-3) = -17}$

-3	4	10	-3	-8
		-12	6	-9
	4	-2	3	-17

Factor Theorem

A polynomial $f(x)$ has a factor $(x-k)$ if and only if $f(x) = 0$.

10. Show that $(x+3)$ is a factor of $f(x) = x^3 \overset{0x^2}{-} 19x - 30$. Then find the remaining factors of $f(x)$.

-3	1	0	-19	-30
		-3	9	30
	1	-3	-10	0

$x^2 - 3x - 10$ $\quad \frac{x}{-10} \quad \frac{+}{-3}$

$x^3 - 19x - 30 = (x+3)(x-5)(x+2)$

remaining factors: $(x-5)(x+2)$

p. 144-145: 9, 13, 16, 29, 31, 33, 39, 47, 49, 55, 58, 59, 60, 63, 67, 69, 75, 81

$(x-2)$ and $(x+3)$ $\quad f(x)=2x^4+7x^3-4x^2-27x-18$

$$\begin{array}{r|rrrr|r} 2 & 2 & 7 & -4 & -27 & -18 \\ & & 4 & 22 & 36 & 18 \\ \hline -3 & 2 & 11 & 18 & 9 & 0 \\ & & -6 & -15 & -9 & \\ \hline & 2 & 5 & 3 & 0 & \end{array}$$

$$2x^2+5x+3$$

$$\begin{array}{|c|c|c} \hline 2x^2 & 2x & 2x \\ \hline 3x & 3 & 3 \\ \hline x & 1 & \end{array}$$

$$f(x)=(x-2)(x+3)(2x+3)(x+1)$$