A ***polynomial function*** is a function in the form $f(x)=a_n x^n + a_{n-1}x^{n-1}+...+a_1 x + a_0$ where a_n, a_{n-1},..., a_1, a_0 are real numbers (coefficients) and n is a nonnegative integer. The domain of a polynomial function is the set of all real numbers. The ***degree*** of a polynomial is the largest power of x that appears. The zero polynomial $f(x)=0$ is not assigned a degree.

$f(x)=3$ degree is 0

Example: Determine which of the following are polynomial functions. For those that are, state the degree; for those that are not, tell why not.

a) $f(x)=5+2x^2-8x^3$ yes ; 3

b) $f(x)=4+3\sqrt{x}$ no ; $\sqrt{\ }$ $\sqrt{x}=x^{\frac{1}{2}}$

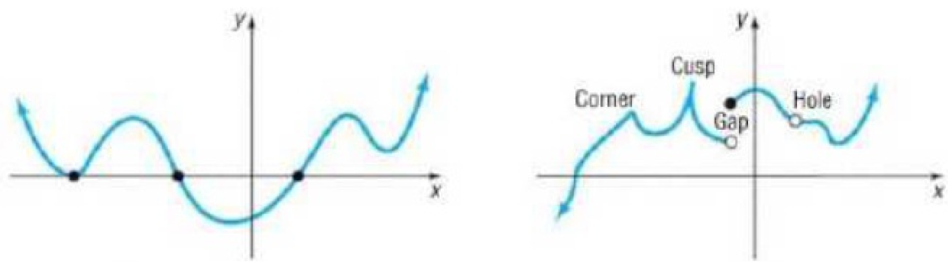
c) $f(x)=-2x^3(x-1)^2$ yes ; 5
$x^3 \cdot x^2$

d) $f(x)=0$ yes ; not assigned

e) $f(x)=\dfrac{x^2-1}{x+4}$ no ; rational

f) $f(x)=9$ yes ; 0

A polynomial function is smooth and continuous. It does not contain corners, cusps, gaps, or holes.

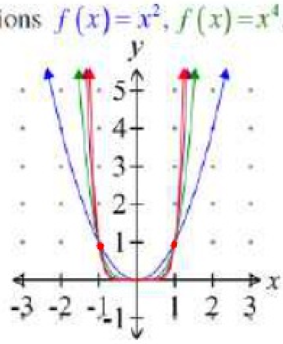


(a) Graph of a polynomial function: smooth, continuous

(b) Cannot be the graph of a polynomial function

A ***power function of degree n*** is a monomial of the form $f(x)=ax^n$, where a is a real number, $a \neq 0$, and n is an integer greater than 0.

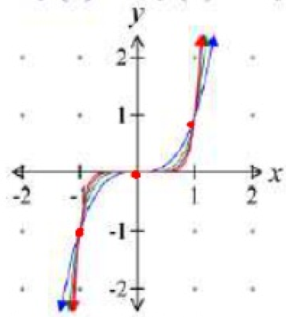
Example: Compare the even power functions $f(x)=x^2$, $f(x)=x^4$, $f(x)=x^6$, and $f(x)=x^8$.



Properties of Power Functions $f(x)=x^n$, n Is an Even Integer

1. f is an even function, so its graph is symmetric with respect to the y-axis.
2. The domain is the set of all real numbers. The range is the set of nonnegative real numbers.
3. The graph always contain the points (-1, 1), (0, 0), and (1, 1).
4. As the exponent n increases in magnitude, the graph becomes more vertical when $x<-1$ or $x>1$; but for x near the origin, the graph tends to flatten out and lie closer to the x-axis.

Example: Compare the odd power functions $f(x)=x^3$, $f(x)=x^5$, $f(x)=x^7$, and $f(x)=x^9$.



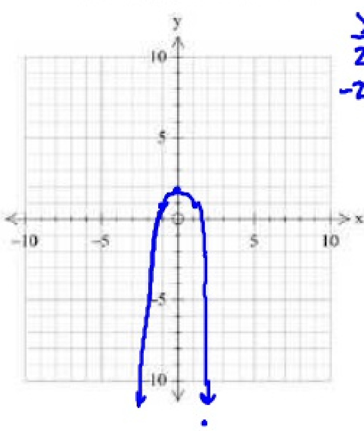
Properties of Power Functions $f(x)=x^n$, n Is an Odd Integer

1. f is an odd function, so its graph is symmetric with respect to the origin.
2. The domain and the range are the set of all real numbers.
3. The graph always contain the points (-1, -1), (0, 0), and (1, 1).
4. As the exponent n increases in magnitude, the graph becomes more vertical when $x<-1$ or $x>1$; but for x near the origin, the graph tends to flatten out and lie closer to the x-axis.

Example: Sketch the graph of each function.

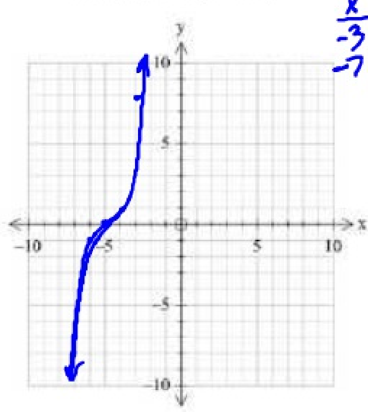
a. $f(x)=-x^4+2$

x	y
2	-14
-2	-14



b. $h(x)=(x+5)^3$

x	y
-3	8
-7	-8

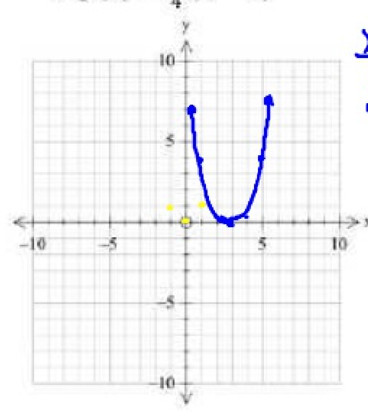


mult. y value by $\frac{1}{4}$

right 3

c. $g(x)=\frac{1}{4}(x-3)^4$

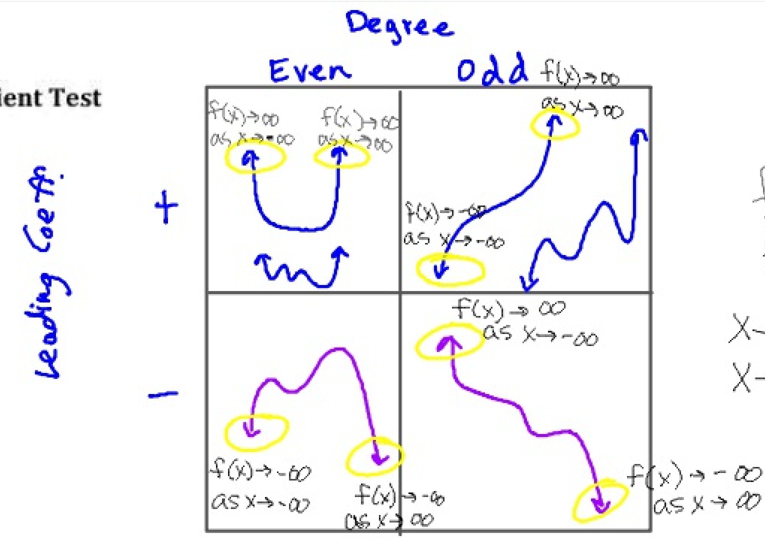
x	y
1	4
5	4



$\frac{1}{4}(1-3)^4$

$\frac{1}{4}(-2)^4=\frac{1}{4}(16)=4$

Leading Coefficient Test



f(x) → ∞ "up"
f(x) → −∞ "down"

x → ∞ right
x → −∞ left

Example: Describe the right-hand and left-hand behavior of the graph of each function.

a. $f(x) = \frac{1}{4}x^3 - 2x$ degree : 3 odd
lead coeff : +

left: $f(x) \to -\infty$ as $x \to -\infty$
right: $f(x) \to \infty$ as $x \to \infty$

b. $f(x) = -3.6x^5 + 5x^3 - 1$
odd deg 5
− lead coeff

left: $f(x) \to \infty$ as $x \to -\infty$
right: $f(x) \to -\infty$ as $x \to \infty$

Real Zeros of Polynomial Functions

If f is a function and r is a real number for which $f(r) = 0$, then r is called a ***real zero*** of f. The real zeros of a polynomial function are the x-intercepts of its graph, and they are found by solving the equation $f(x) = 0$.

The following statements are equivalent:

1. r is a solution of $f(x) = 0$.
2. r is a real zero of a polynomial function f.
3. r is an x-intercept of the graph of f.
4. $x - r$ is a factor of f.

Example: Find all zeros of $f(x) = x^3 - 12x^2 + 36x$. Then determine the maximum possible number of turning points of the graph of the function.

one less than the power

max turning points: $\boxed{2}$

factor to find the zeros

$f(x) = x(x^2 - 12x + 36)$ $\frac{\times}{36}$ $\frac{+}{-12}$

$= x(x-6)(x-6)$

$= x(x-6)^2 = 0$

$x = 0$

$x - 6 = 0$
$+6 \quad +6$
$x = 6$

$\boxed{\text{zeros: } 0, 6}$ repeated 0

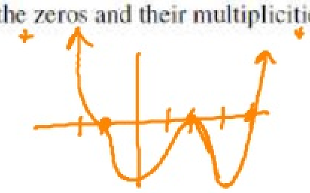
(0,0) (6,0)

$x = 0, x = 6$

If the same factor $x - r$ occurs more than once, r is called a ***repeated***, or ***multiple zero*** **of** f. More precisely, if $(x-r)^m$ is a factor of a polynomial f and $(x-r)^{m+1}$ is not a factor of f, then r is called a ***zero of multiplicity m*** **of** f.

Example: Identify the zeros and their multiplicities for the polynomial $f(x) = 3(x-4)(x+1)^3(x-2)^2$.

degree : 6
lead. coeff : +



$x - 4 = 0$
$x = 4$
cut through

$x + 1 = 0$
$x = -1$
twist through

$x - 2 = 0$
$x = 2$
bounce

Multiplicity Rules:

If r is a zero of even multiplicity:

1. The sign of $f(x)$ does not change from one side to the other side of r.
2. The graph of f **touches** the x-axis at r. bounce

If r is a zero of odd multiplicity:

1. the sign of $f(x)$ changes from one side to the other of r.
2. The graph of f **crosses** the x-axis at r. through

Example: Sketch the graph of $f(x) = 2x^3 - 6x^2$.

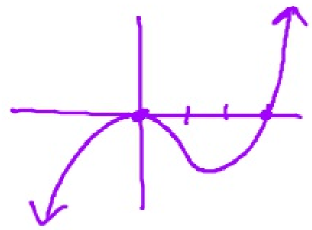
$f(x) = 2x^2(x-3)$

$2x^2(x-3)=0$

$2x^2=0 \qquad x-3=0$

$x=0 \qquad x=3$

bounce   through



Example: Sketch the graph of $f(x) = -\frac{1}{4}x^4 + \frac{3}{2}x^3 - \frac{9}{4}x^2$.

$\frac{3}{2} \cdot -\frac{4}{1}$

$f(x) = -\frac{1}{4}x^2(x^2-6x+9)$

$\frac{\times}{9} \quad \frac{+}{-6}$

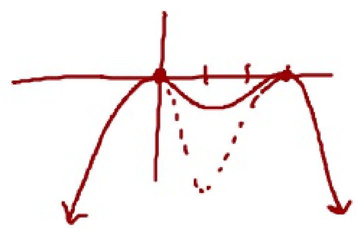
$-3, -3$

$= -\frac{1}{4}x^2(x-3)^2$

$\downarrow \qquad \downarrow$

$0 \qquad 3$

bounce   bounce



Intermediate Value Theorem

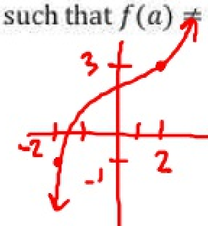
Let a and b be real numbers such that $a < b$. If f is a polynomial (continuous) function such that $f(a) \neq f(b)$, then, in the interval $[a,b]$, f takes on every value between $f(a)$ and $f(b)$.

$\downarrow$

f(x) or y

$a=-2 \qquad f(a)=-1$

$b=2 \qquad f(b)=3$



p. 133: 9-14, 15 – 29 odd, 31, 35, 37, 41, 47, 51, 53, 55, 57, 61, 65, 67, 69, 71, 73, 75, 79, 83, 89, 97

p.132 $f(x)=x^3-x^2+1$

x	-2	-1	0	1
f(x)	-11	-1	1	1

$\downarrow$

0