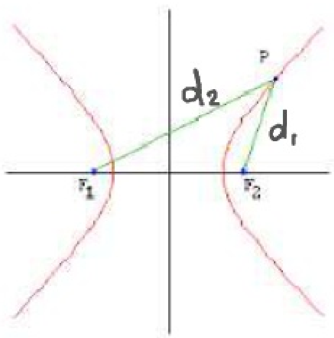
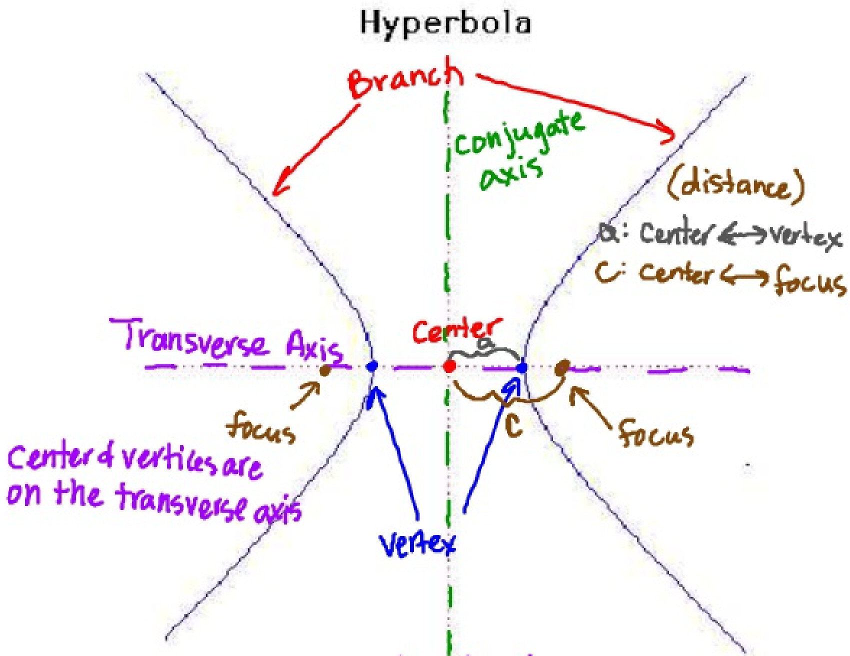
A **hyperbola** is the set of all points (x, y) in a plane for which the absolute value of the difference of the distances from two distinct fixed points, called foci, is constant.

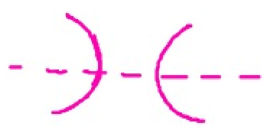


$|d_2 - d_1|$ is constant



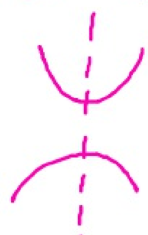
Standard Equation of a hyperbola:

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

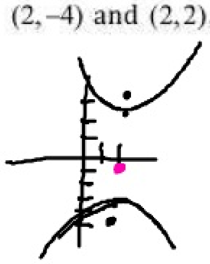


$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$c^2 = a^2 + b^2$$

Examples:

1. Find the standard form of the equation of the hyperbola having foci $(2, -5)$ and $(2, 3)$ and vertices $(2, -4)$ and $(2, 2)$.



center $(2, -1)$

$a = 3$
$c = 4$

$(4)^2 = (3)^2 + b^2$
$16 = 9 + b^2$
$7 = b^2$

$$\boxed{\frac{(y+1)^2}{9} - \frac{(x-2)^2}{7} = 1}$$

Asymptotes of a hyperbola

Each hyperbola has two **asymptotes** that intersect at the center of the hyperbola. The asymptotes pass through the vertices of a rectangle of dimensions $2a$ by $2b$, with its center at (h, k).

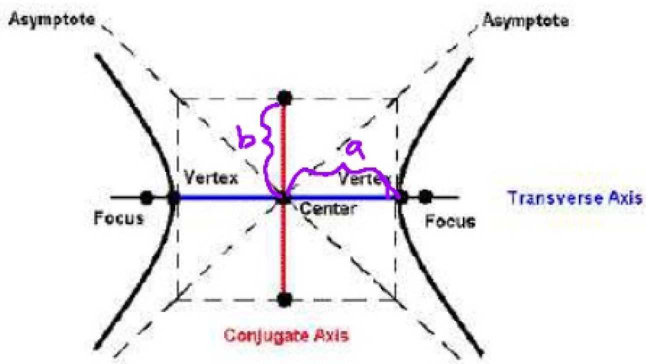
Equations of Asymptotes:

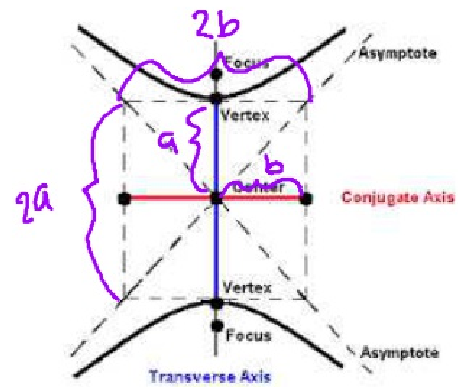
Horizontal Transverse Axis

$y = k \pm \frac{b}{a}(x-h)$

Vertical Transverse Axis

$y = k \pm \frac{a}{b}(x-h)$



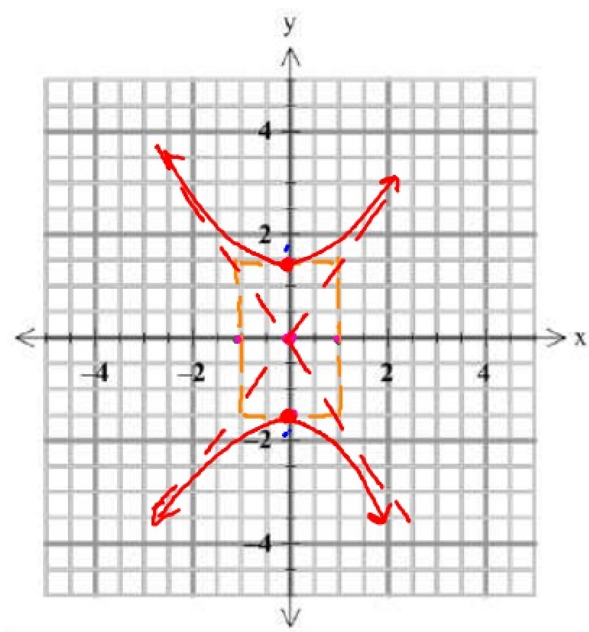


2. Sketch the hyperbola $\frac{4y^2}{36} - \frac{9x^2}{36} = \frac{36}{36}$.

$\frac{y^2}{9} - \frac{x^2}{4} = 1$

center: $(0,0)$ $a=3$ $b=2$

asymptotes: $y = \pm \frac{3}{2}x$

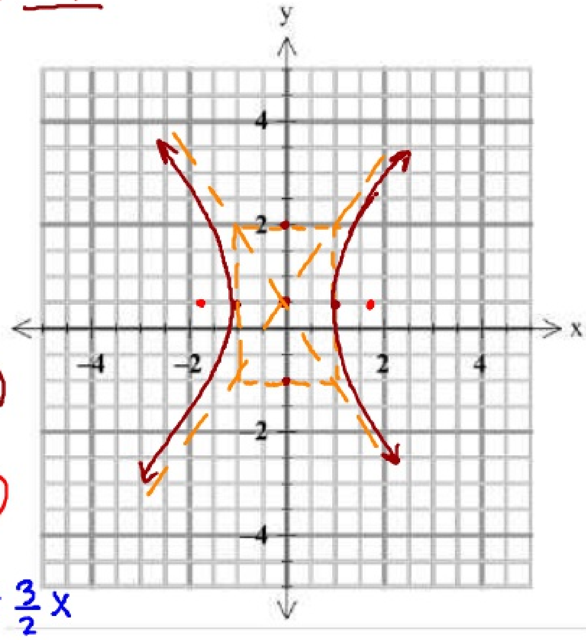


3. Sketch the hyperbola $9x^2 - 4y^2 + 8y - 40 = 0$, find the equations of its asymptotes, and find the foci.

$$9x^2 - 4(y^2 - 2y + \underline{1}) = 40 + \underline{-4}$$

$$\frac{9x^2}{36} - \frac{4(y-1)^2}{36} = \frac{36}{36}$$

$$\frac{x^2}{4} - \frac{(y-1)^2}{9} = 1$$



Center: $(0,1)$

Vertices: $(2,1), (-2,1)$

$a = 2$

$b = 3$

Foci: $(\sqrt{13}, 1), (-\sqrt{13}, 1)$

$c^2 = a^2 + b^2$

$c^2 = 4 + 9$

$c^2 = 13$ $\quad c = \sqrt{13}$

asymptotes: $y = 1 \pm \frac{3}{2}x$

4. Find the standard form of the equation of the hyperbola having vertices $(3,2)$ and $(9,2)$ and having asymptotes $y = -2 + \frac{2}{3}x$ and $y = 6 - \frac{2}{3}x$

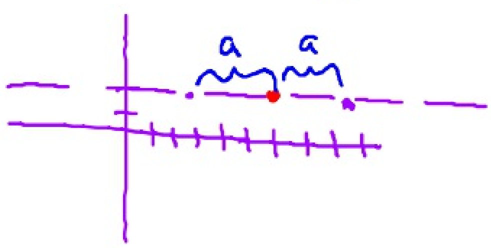
b

a

Center: $(6,2)$

$a = 3$

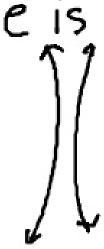
$b = 2$



$$\boxed{\frac{(x-6)^2}{9} - \frac{(y-2)^2}{4} = 1}$$

Eccentricity: $e = \frac{c}{a}$

e is large



e is small

e closer to 1

Classifying a Conic from its General Equation

The graph of $Ax^2 + Cy^2 + Dx + Ey + F = 0$ is one of the following.

1. *Circle:* $A = C$ $A \neq 0$ & $C \neq 0$ $2x^2 + 2y^2 = 4$
2. *Parabola:* $AC = 0$ $A = 0$ or $C = 0$ but <u>not</u> <u>both</u>
3. *Ellipse:* $AC > 0$ A & C have the same signs
4. *Hyperbola:* $AC < 0$ A & C have opposite signs

Example:

6. Classify the graph of each equation.

a. $3x^2 + 3y^2 - 6x + 6y + 5 = 0$

Circle $A = C = 3$

b. $2x^2 - 4y^2 + 4x + 8y - 3 = 0$

Hyperbola $A \cdot C < 0$
$(2)(-4) < 0$
$-8 < 0$

c. $3x^2 + y^2 + 6x - 2y + 3 = 0$

Ellipse $A \cdot C > 0$
$(3)(1) > 0$
$3 > 0$

d. $2x^2 + 4x + y - 2 = 0$

Parabola $A \cdot C = 0$
$(2)(0) = 0$

p. 720: 5-8, 9, 11, 15, 19, 25, 31, 39, 43, 45, 53 – 60, 76