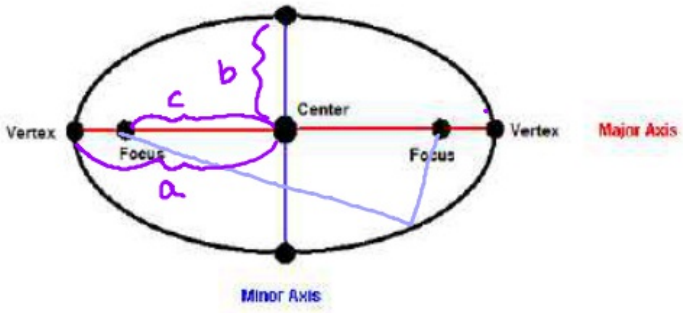
ELLIPSES **SECTION 10.3** **NOTES**

An **ellipse** is the set of all points (x, y) in a plane, the sum of whose distances from the two distinct fixed points, called foci, is constant.

the longer part is on the major axis



Standard Equation of an Ellipse:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$ horizontal major axis

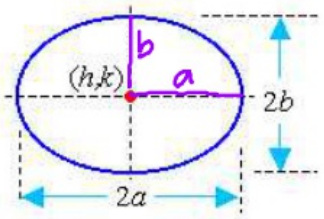
$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$ vertical major axis

$a > b$

If $a = b$, then you have a circle.

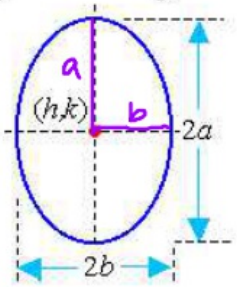
Ellipse type 1:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$



Ellipse type 2:

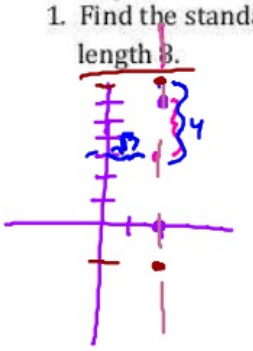
$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$



$b^2 = a^2 - c^2$ or $b = \sqrt{a^2 - c^2}$

Examples:

1. Find the standard form of the equation of the ellipse having foci $(2,0)$ and $(2,6)$ and a major axis of length 8.



center $(2,3)$

$c = 3$

$a = 4 \quad a^2 = 16$

$b^2 = 4^2 - 3^2$

$b^2 = 16 - 9$

$b^2 = 7$

$\frac{(x-2)^2}{b^2} + \frac{(y-3)^2}{a^2} = 1$

$\frac{(x-2)^2}{7} + \frac{(y-3)^2}{16} = 1$

2. Find the center, vertices, and foci of the ellipse $9x^2 + 4y^2 + 36x - 8y + 4 = 0$. Then sketch the ellipse.

$(9x^2 + 36x + \_\_) + (4y^2 - 8y + \_\_) = -4$

$9(x^2 + 4x + \underline{4}) + 4(y^2 - 2y + \underline{1}) = -4 + \underline{36} + \underline{4}$

$\frac{9(x+2)^2}{36} + \frac{4(y-1)^2}{36} = \frac{36}{36}$

$\frac{(x+2)^2}{4} + \frac{(y-1)^2}{9} = 1$

$b = 2 \quad a = 3$

center $(-2,1)$

vertices $(-2,4)$ $(-2,-2)$

foci

$4 = 9 - c^2$

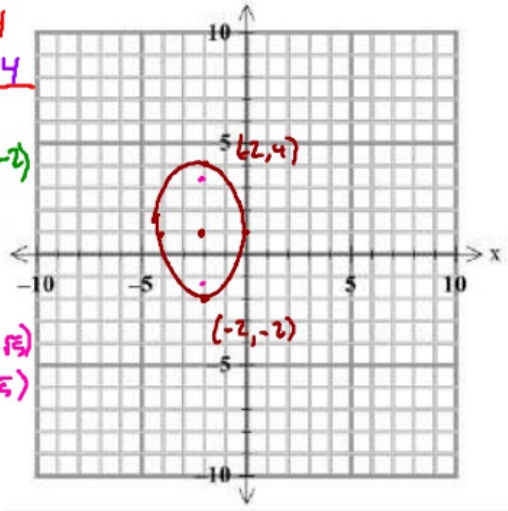
$-5 = -c^2$

$5 = c^2$

$c = \sqrt{5}$

$(-2, 1+\sqrt{5})$

$(-2, 1-\sqrt{5})$



3. Find the center, vertices, and foci of the ellipse $5x^2 + 9y^2 + 10x - 54y + 41 = 0$. Then sketch the ellipse.

$(5x^2 + 10x + \_\_) + (9y^2 - 54y + \_\_) = -41$

$5(x^2 + 2x + \underline{1}) + 9(y^2 - 6y + \underline{9}) = -41 + \underline{5} + \underline{81}$

$\frac{5(x+1)^2}{45} + \frac{9(y-3)^2}{45} = \frac{45}{45}$

$\frac{(x+1)^2}{9} + \frac{(y-3)^2}{5} = 1$

$b^2 = 5$

$b = \sqrt{5}$

$a^2 = 9$

$a = 3$

$5 = 9 - c^2$

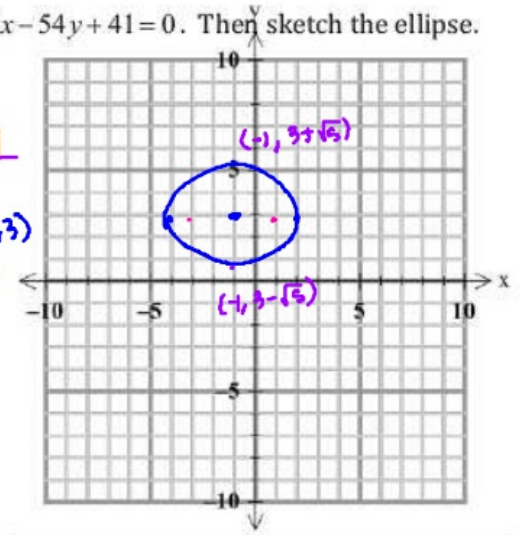
$c^2 = 4$

$c = 2$

center: $(-1,3)$

vertices: $(-4,3)$ $(2,3)$

foci: $(-3,3)$ $(1,3)$



Eccentricity: $e = \frac{c}{a}$ (ovalness)

p. 710: 5-8, 9, 11, 13, 17, 19, 21, 23, 25, 33, 37, 41, 45, 55