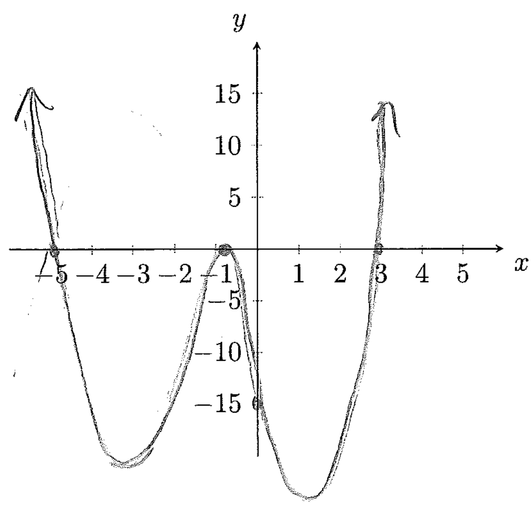
Review for Midterm #2

Math 1050 Instructor: Louise Atkinson

Name: Key

Polynomial Functions

1. For $g(x) = x^4 + 4x^3 - 10x^2 - 28x - 15$ (hint: $x = 3, x = -1$ are roots) find:



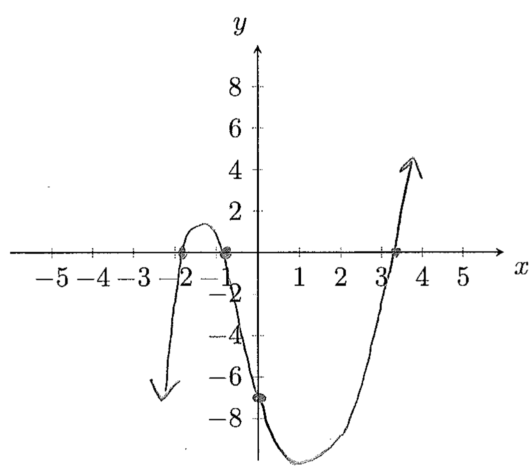
1. All the factors: $\boxed{(x+5)(x+1)^2(x-3)}$
2. x intercept(s): $\boxed{(-5,0)\ (-1,0)\ (3,0)}$
3. y intercept: $\boxed{(0,-15)}$
4. Where $g(x) > 0$ $\boxed{(-\infty,-5)\cup(3,\infty)}$
5. As $x \to \infty, g(x) \to$ $\boxed{\infty}$
6. As $x \to -\infty, g(x) \to$ $\boxed{\infty}$
7. Graph

$3 \,|\ 1 \quad 4 \quad -10 \quad -28 \quad -15$
$\quad\ \ 3 \quad 21 \quad 33 \quad 15$
$-1 \,|\ 1 \quad 7 \quad 11 \quad 5 \,|\, 0$
$\quad\ \ -1 \quad -6 \quad -5$
$1 \quad 6 \quad 5 \,|\, 0$

$x^2 + 6x + 5$

$(x+5)(x+1)$

2. For $h(x) = x^3 - 8x - 7$ (hint: $(x+1)$ is a factor) find:



1. All the factors: $(x+1)\left(x - \frac{1+\sqrt{29}}{2}\right)\left(x - \frac{1-\sqrt{29}}{2}\right)$
2. x intercept(s): $(-1,0), \left(\frac{1+\sqrt{29}}{2}, 0\right) \left(\frac{1-\sqrt{29}}{2}, 0\right)$ 3.19 -2.19
3. y intercept: $(0,-7)$
4. Where $h(x) > 0$ $\left(\frac{1-\sqrt{29}}{2}, -1\right)\cup\left(\frac{1+\sqrt{29}}{2}, \infty\right)$
5. As $x \to \infty, h(x) \to$ ∞
6. As $x \to -\infty, h(x) \to$ $-\infty$
7. Graph

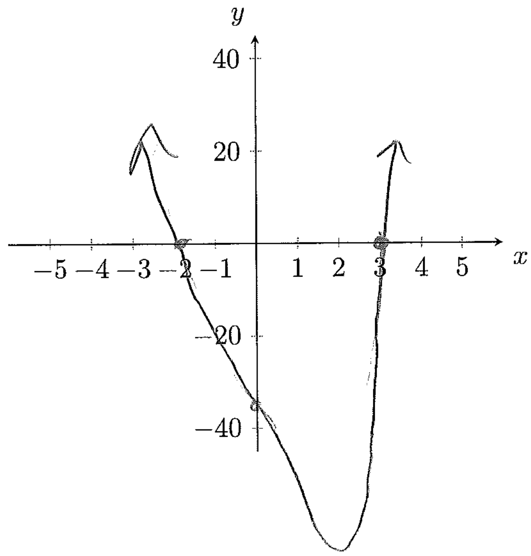
$-1 \,|\ 1 \quad 0 \quad -8 \quad -7$
$\quad\ \ -1 \quad 1 \quad 7$
$1 \quad -1 \quad -7 \,|\, 0$

$x^2 - x - 7$

$$x = \frac{1 \pm \sqrt{1+28}}{2}$$

$$\frac{1 \pm \sqrt{29}}{2}$$

3. For $f(x) = x^4 + 3x^3 - 4x^2 - 30x - 36$ (hint: $x = 3, x = -2$ are solutions) find:



1. All the factors: $(x-3)(x+2)$
2. x intercept(s): $(3,0)\ (-2,0)$
3. y intercept: $(0,-36)$
4. Where $f(x) > 0$
5. As $x \to \infty, f(x) \to$ ∞
6. As $x \to -\infty, f(x) \to$ ∞
7. Graph

$$\begin{array}{r|rrrrr} -2 & 1 & 3 & -4 & -30 & -36 \\ & & -2 & -2 & 12 & 36 \\ \hline 3 & 1 & 1 & -6 & -18 & 0 \\ & & 3 & 12 & 18 & \\ \hline & 1 & 4 & 6 & 0 & \end{array}$$

x^2+4x+6

$x = \frac{-4 \pm \sqrt{16-24}}{2}$

$x = \frac{-4 \pm \sqrt{-8}}{2}$

$x = \frac{-4 \pm 2i\sqrt{2}}{2}$

$x = -2 \pm i\sqrt{2}$

4. For $f(x) = x^3 - 2x^2 - 2x + 1$ (hint: $(x+1)$ is a factor), sketch the graph and show the x and y intercepts.

x-int $(-1,0)$

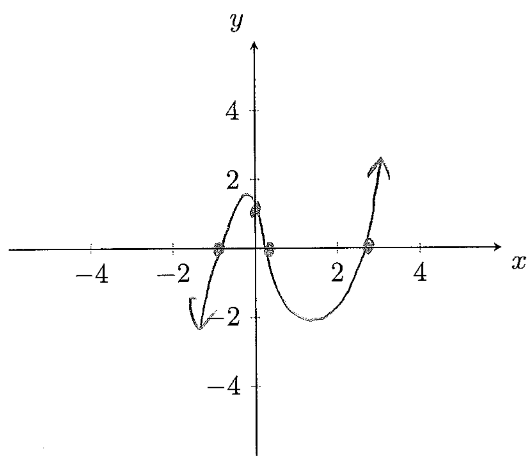
$\left(\frac{3-\sqrt{5}}{2}, 0\right)$

0.38

$\left(\frac{3+\sqrt{5}}{2}, 0\right)$

2.62

y-int $(0,1)$



$$\begin{array}{r|rrrr} -1 & 1 & -2 & -2 & 1 \\ & & -1 & 3 & -1 \\ \hline & 1 & -3 & 1 & 0 \end{array}$$

x^2-3x+1

$x = \frac{3 \pm \sqrt{9-4}}{2}$

$x = \frac{3 \pm \sqrt{5}}{2}$

$\approx 0.38, 2.62$

5. State all the possible rational roots for:

(a) $f(x) = 4x^4 + 3x^3 - 2x^2 + 10x - 18$ $\boxed{\pm\{1, 2, 3, 6, 9, 18, \frac{1}{2}, \frac{3}{2}, \frac{9}{2}, \frac{1}{4}, \frac{3}{4}, \frac{9}{4}\}}$

(b) $f(x) = 5x^4 + 2x^3 - 6x^2 + 4x - 8$ $\boxed{\pm\{1, 2, 4, 8, \frac{1}{5}, \frac{2}{5}, \frac{4}{5}, \frac{8}{5}\}}$

a) $p: \pm\{1, 2, 3, 6, 9, 18\}$

$q: \pm\{1, 2, 4\}$

b) $p: \pm\{1, 2, 4, 8\}$

$q: \pm\{1, 5\}$

6. Find a third degree polynomial $f(x)$ that has a real root at $x = 1$ and a complex root at $x = 1 + i$.

$f(x) = (x-1)(x-(1+i))(x-(1-i))$
$= (x-1)(x-1-i)(x-1+i)$
$= (x-1)(x^2-2x+2)$

	x	-1	$-i$
x	x^2	$-x$	$\cancel{-ix}$
-1	$-x$	1	$\cancel{i}$
i	$\cancel{ix}$	$\cancel{-i}$	$-i^2$

	x^2	$-2x$	2
x	x^3	$-2x^2$	$2x$
-1	$-x^2$	$2x$	-2

$\boxed{f(x) = x^3-3x^2+4x-2}$

7. Find a third degree polynomial $g(x)$ that has a real root at $x = -1$ and a complex root at $x = 1 - 3i$.

$g(x) = (x+1)(x-(1-3i))(x-(1+3i))$
$= (x+1)(x-1+3i)(x-1-3i)$
$= (x+1)(x^2-2x+10)$

	x	-1	$3i$
x	x^2	$-x$	$\cancel{3ix}$
-1	$-x$	1	$\cancel{-3i}$
$-3i$	$\cancel{-3ix}$	$\cancel{3i}$	$-9i^2$

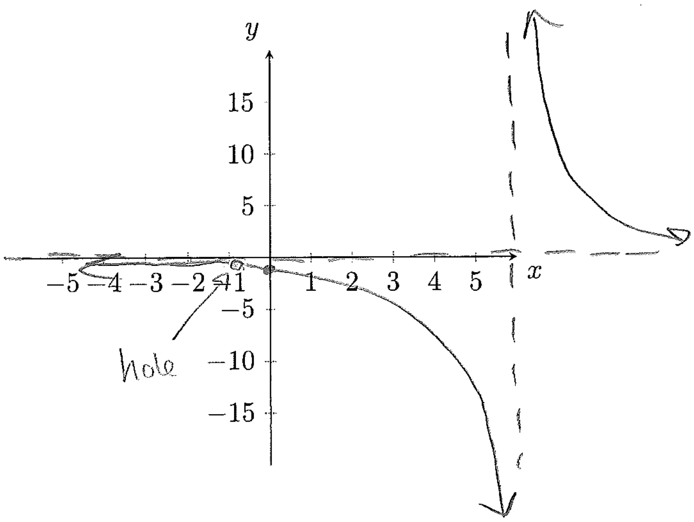
	x^2	$-2x$	10
x	x^3	$-2x^2$	$10x$
1	x^2	$-2x$	10

$\boxed{g(x) = x^3-x^2+8x+10}$

Rational Functions

8. Find the following information for the given function:

$$g(x) = \frac{x+1}{x^2-5x-6} = \frac{\cancel{x+1}}{\cancel{(x+1)}(x-6)}$$ hole at $x=-1$



1. VA(s): $x = 6$
2. HA (end behavior): $y = 0$
3. x intercept(s): none
4. y intercept(s): $(0, -\frac{1}{6})$
5. Graph
6. Domain: $(-\infty,-1)\cup(-1,6)\cup(6,\infty)$

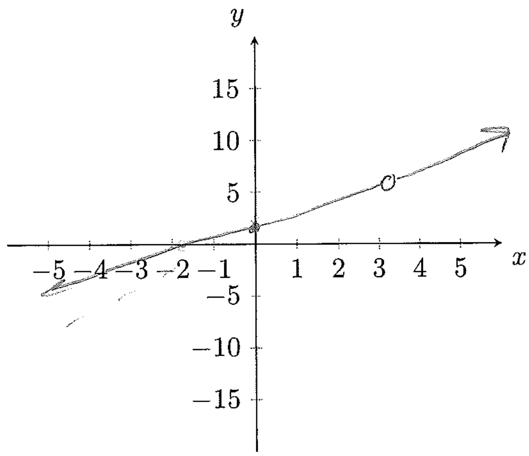
$g(x) = \frac{1}{x-6} \rightarrow$

$g(-1) = -\frac{1}{7}$

9. Find the following information for the given function:

$$g(x) = \frac{x^2 - x - 6}{x - 3} = \frac{\cancel{(x-3)}(x+2)}{\cancel{x-3}}$$

hole at $x = 3$

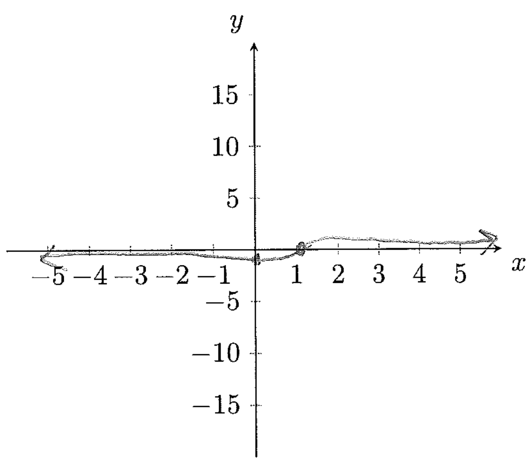


1. VA(s): none
2. HA (end behavior): HA none (End $y = x + 2$)
3. x intercept(s): $(-2, 0)$
4. y intercept(s): $(0, 2)$
5. Graph
6. Domain: $(-\infty, 3) \cup (3, \infty)$

$$\begin{array}{r} x + 2 \\ x-3 \overline{) x^2 - x - 6} \\ -x^2 + 3x \\ \hline 2x - 6 \end{array}$$

10. Find the following information for the given function:

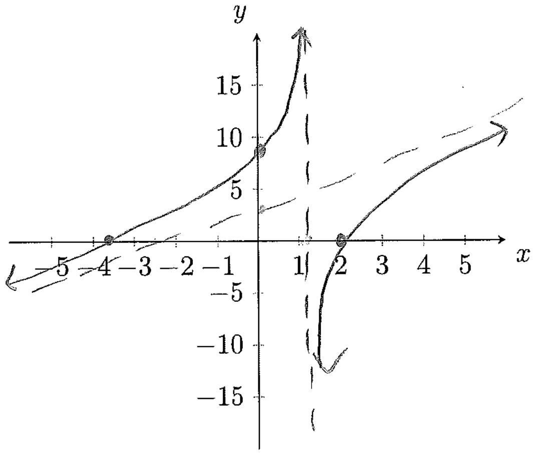
$$g(x) = \frac{x - 1}{x^2 + 9}$$



1. VA(s): none
2. HA (end behavior): $y = 0$
3. x intercept(s): $(1, 0)$
4. y intercept(s): $(0, -\frac{1}{9})$
5. Graph
6. Domain: $(-\infty, \infty)$

11. Find the following information for the given function:

$$g(x) = \frac{x^2+2x-8}{x-1} = \frac{(x+4)(x-2)}{x-1}$$



1. VA(s): $x=1$

2. HA (end behavior): none HA (End $y=x+3$)

$$\begin{array}{r} x+3 \\ x-1 \,\overline{)\, x^2+2x-8} \\ -x^2+x \\ \hline 3x-8 \\ -3x+3 \\ \hline -5 \end{array}$$

3. x intercept(s): $(-4,0)\ (2,0)$

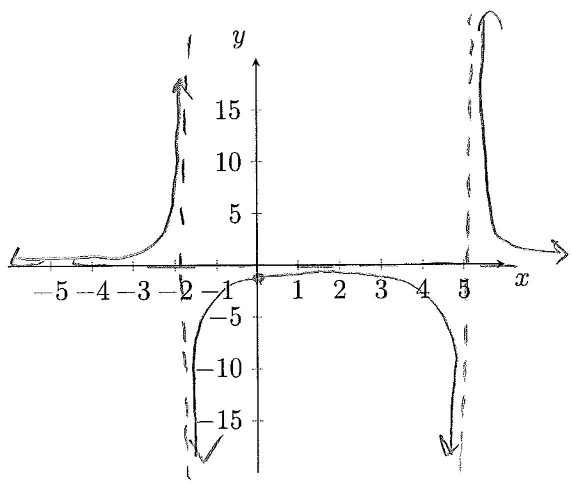
4. y intercept(s): $(0,8)$

5. Graph

6. Domain: $(-\infty,1)\cup(1,\infty)$

12. Find the following information for the given function:

$$g(x) = \frac{4}{x^2-3x-10} = \frac{4}{(x-5)(x+2)}$$



1. VA(s): $x=5,\ x=-2$

2. HA (end behavior): $y=0$

3. x intercept(s): none

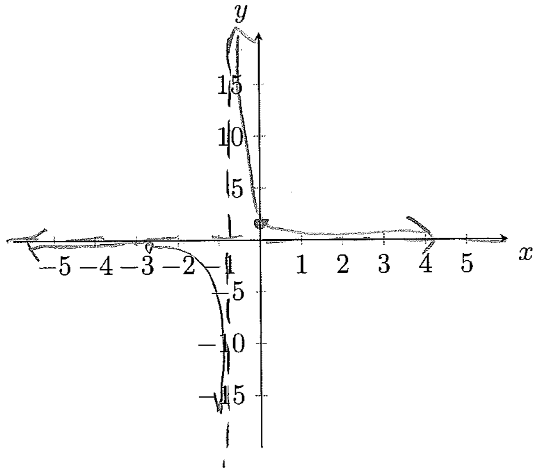
4. y intercept(s): $(0,-\frac{2}{5})$

5. Graph

6. Domain: $(-\infty,-2)\cup(-2,5)\cup(5,\infty)$

13. Find the following information for the given function:

$$g(x) = \frac{x+3}{x^2+4x+3} = \frac{(x+3)}{(x+1)(x+3)} = \frac{1}{x+1}$$ hole @ x = -3



1. VA(s): X = -1

2. HA (end behavior): y = 0

3. x intercept(s): none

4. y intercept(s): (0, 1)

5. Graph

6. Domain: $(-\infty,-3) \cup (-3,-1) \cup (-1,\infty)$

14. State the end behavior for the following polynomials:

(a) $f(x) = 4 + 3x^2 - x^3$

- As $x \to \infty$, $f(x) \to$ $-\infty$
- As $x \to -\infty$, $f(x) \to$ ∞

(b) $f(x) = -5 - x + 3x^2 + x^4$

- As $x \to \infty$, $f(x) \to$ ∞
- As $x \to -\infty$, $f(x) \to$ ∞

(c) $f(x) = -x(x+2)(x-2)^3(x+5)^2(x+1)^8$

degree: 15
lead coeff: −

↑ ↓

- As $x \to \infty$, $f(x) \to$ $-\infty$
- As $x \to -\infty$, $f(x) \to$ ∞

(d) $f(x) = 4x - x^3 - 5x^4 + x^2$

- As $x \to \infty$, $f(x) \to$ $-\infty$
- As $x \to -\infty$, $f(x) \to$ $-\infty$

15. Solve each of the following:

(a) $x^2 - 3x > 18$

(b) $x^2 - 6x + 9 < 16$

(c) $4x^3 - 12x^2 < 0$

(d) $5x(x-1)(x+3)^2(x+1)^3 > 0$

(e) $2x(x-3)^3(x+1)^2(2-x) > 0$

Have not learned yet

(f) $\dfrac{1}{3x-1} < 2$

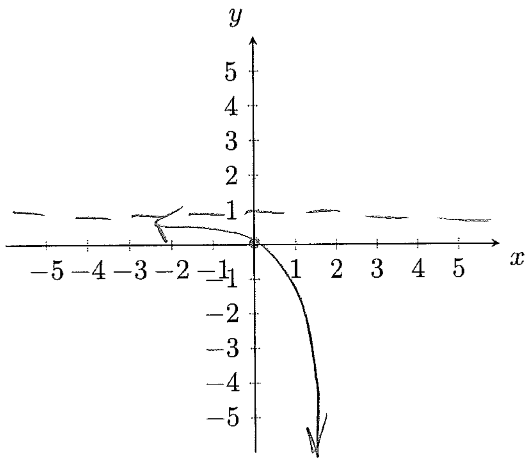
(g) $\dfrac{4x-1}{x} > 0$

(h) $\dfrac{x+1}{x-3} > 2$

(i) $\dfrac{x-2}{x-1} > \dfrac{x-1}{x+2}$

16. Graph each of the following functions. State the domain, range and any asymptotes.

(a) $f(x) = -e^x + 1$

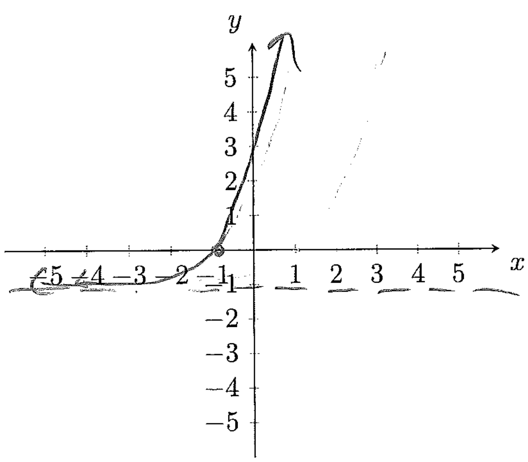


(a) Domain: $(-\infty, \infty)$

(b) Range: $(-\infty, 1)$

(c) Asymptotes: $y = 1$

(b) $f(x) = e^{x+1} - 1$

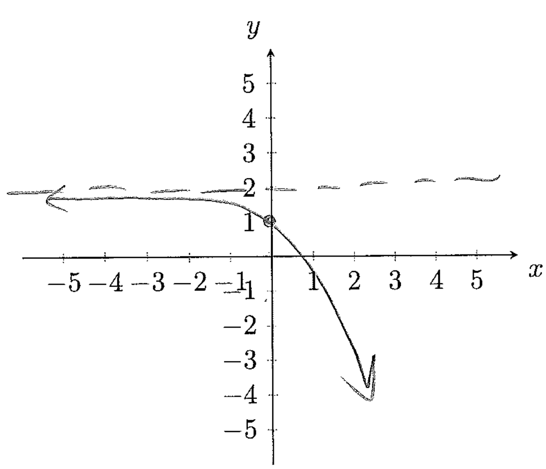


(a) Domain: $(-\infty, \infty)$

(b) Range: $(-1, \infty)$

(c) Asymptotes: $y = -1$

(c) $f(x) = 2 - e^x$

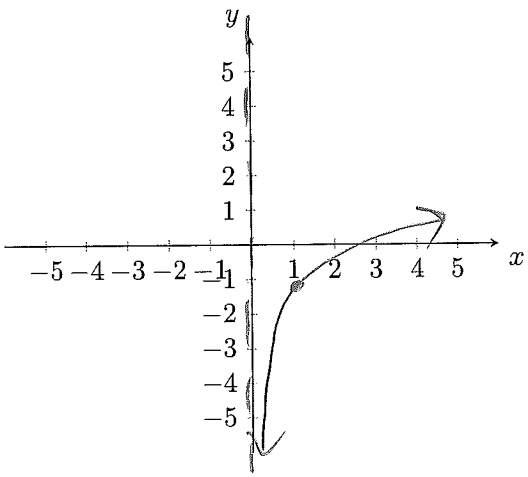


(a) Domain: $(-\infty, \infty)$

(b) Range: $(-\infty, 2)$

(c) Asymptotes: $y = 2$

(d) $f(x) = \ln x - 1$

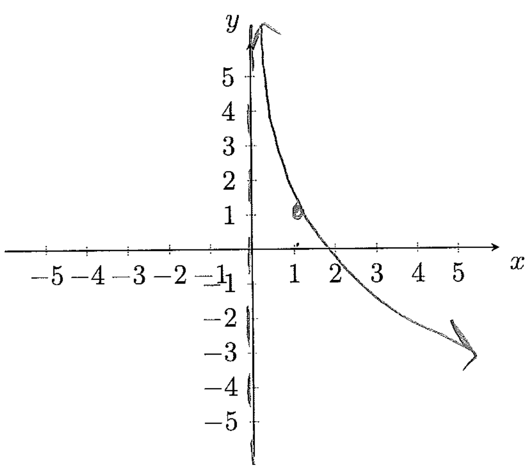


(a) Domain: $(0, \infty)$

(b) Range: $(-\infty, \infty)$

(c) Asymptotes: $x = 0$

(e) $f(x) = -\ln x + 1$

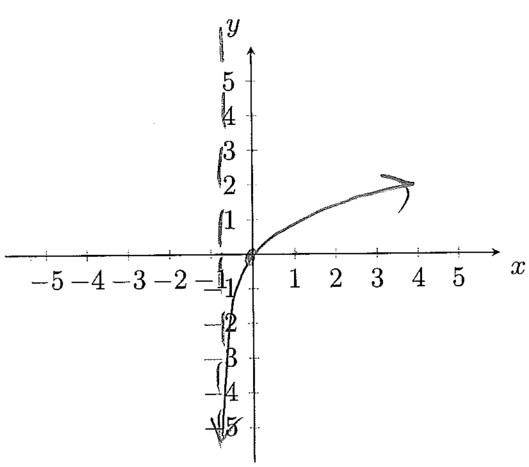


(a) Domain: $(0, \infty)$

(b) Range: $(-\infty, \infty)$

(c) Asymptotes: $x = 0$

(f) $f(x) = \ln(x + 1)$



(a) Domain: $(-1, \infty)$

(b) Range: $(-\infty, \infty)$

(c) Asymptotes: $x = -1$

17. Simplify the following to a rational number

(a) $\log_4 32$ $\quad 2^{2x} = 2^5$ $\quad \boxed{5/2}$

(b) $\log_5 0$ $\quad$ NA can't take log of 0

(c) $\ln e$ $= \boxed{1}$

(d) $\ln 1$ $= \boxed{0}$

(e) $\log_{\frac{1}{25}} 125$ $\quad 5^{-2x} = 5^3$ $\quad \boxed{-\frac{3}{2}}$

(f) $\log_{\frac{1}{9}} 27$ $\quad 3^{-2x} = 3^3$ $\quad \boxed{-\frac{3}{2}}$

(g) $\log_{\frac{1}{36}} 1$ $= \boxed{0}$

(h) $\ln \frac{1}{\sqrt{e}}$ $= \ln e^{-\frac{1}{2}} = \boxed{-\frac{1}{2}}$

(i) $2 \ln e^6 - \ln e^5$ $= \ln \frac{e^{12}}{e^5} = \ln e^7 = \boxed{7}$

18. Use the rules of logs to expand:

(a) $\ln \sqrt[3]{xy^2}$ $= \ln(x^{\frac{1}{3}} y^{\frac{2}{3}}) = \frac{1}{3}\ln x + \frac{2}{3}\ln y$

(b) $\log \frac{\sqrt{x}}{3y^2}$ $= \frac{1}{2}\log x - (\log 3 + 2\log y)$

OR $\frac{1}{2}\log x - \log 3 - 2\log y$

19. Write each expression as a log of a single quality:

(a) $2\log x - 3\log y$ $\quad \log \frac{x^2}{y^3}$

(b) $3\log(x+2) - (\frac{1}{2}\log y + 2\log(x-1))$ $\quad \log \frac{(x+2)^2}{\sqrt{y+2}\,(x-1)^2}$

20. Solve for x in the following problems:

(a) $27^{x-1} = \left(\frac{1}{9}\right)^{5-2x}$

(b) $(4^{x-1})(2^x) = \left(\frac{1}{16}\right)^{3x+1}$

(c) $\ln(2x-5) = \ln 7$

(d) $\log_3(4-7x) = \log_3(x+2)$

a) $(3^3)^{(x-1)} = (3^{-2})^{(5-2x)}$

$3^{3x-3} = 3^{-10+4x}$

$3x-3 = -10+4x$

$\boxed{7 = x}$

b) $(2^{2x-2})(2^x) = (2^{-4})^{(3x+1)}$

$2^{3x-2} = 2^{-12x-4}$

$3x-2 = -12x-4$

$15x = -2$

$\boxed{x = -\frac{2}{15}}$

c) $2x-5=7$

$2x=12$

$\boxed{x=6}$

d) $4-7x=x+2$

$2=8x$

$\boxed{x=\frac{1}{4}}$