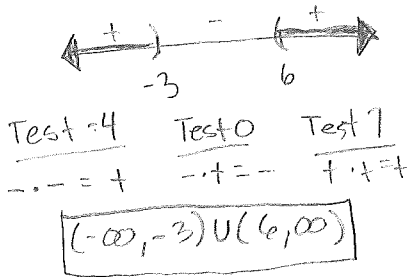


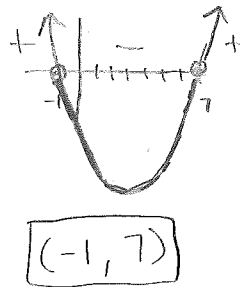
Review for Midterm #3

Solutions

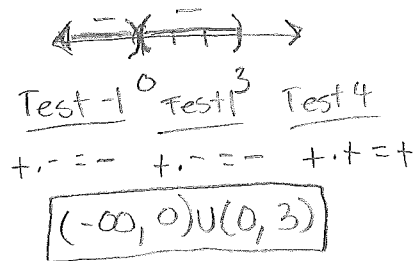
1. $x^2 - 3x > 18$
 $x^2 - 3x - 18 > 0$
 $(x-6)(x+3) > 0$



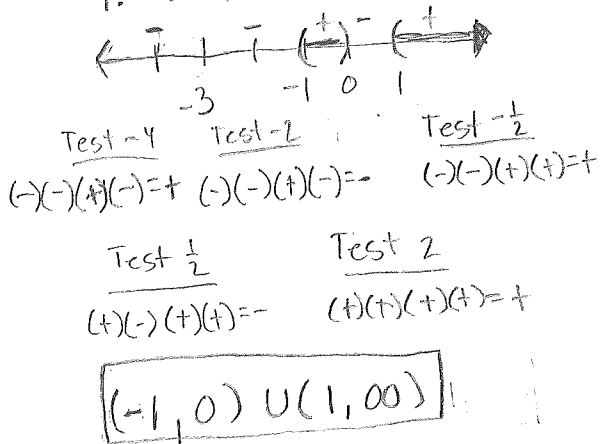
2. $x^2 - 6x + 9 < 16$
 $x^2 - 6x - 7 < 0$
 $(x-7)(x+1) < 0$



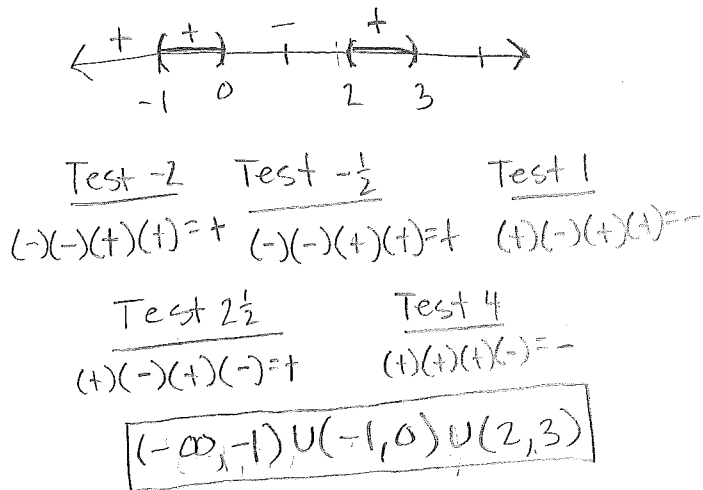
3. $4x^3 - 12x^2 < 0$
 $4x^2(x-3) < 0$



4. $5x(x-1)(x+3)(x+1)^3 > 0$



5. $2x(x-3)^3(x+1)^2(2-x) > 0$

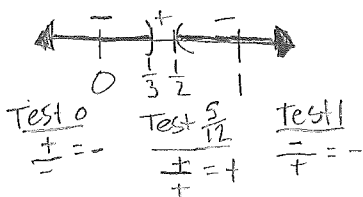


6. $\frac{1}{3x-1} < 2$ $(-\infty, \frac{1}{3}) \cup (\frac{1}{2}, \infty)$

$\frac{1}{3x-1} - 2 < 0$
 $\frac{1 - 6x + 2}{3x-1} < 0$

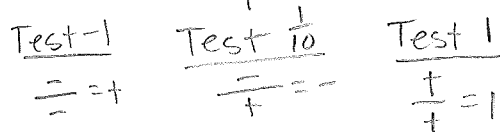
$\frac{-6x+2}{3x-1} < 0$

$\frac{-6x+3}{3x-1} < 0$



$-6x+3=0$
 $3=6x$
 $\frac{1}{2}=x$
 $3x-1=0$
 $3x=1$
 $x=\frac{1}{3}$

7. $\frac{4x-1}{x} > 0$ $4x-1=0$
 $x=\frac{1}{4}$



Test: $(-\infty, 0) \cup (1/4, \infty)$

$$8. \frac{x+1}{x-3} > 2$$

$$\frac{x+1}{x-3} - 2 \cdot \frac{(x-3)}{(x-3)} > 0$$

$$\frac{x+1-2x+6}{x-3} > 0$$

$$\frac{-x+7}{x-3} > 0$$



Test 0	Test 4	Test 8
$\frac{+}{-} = -$	$\frac{+}{+} = +$	$\frac{-}{+} = -$

$$(3, 7)$$

$$9. \frac{x-2}{x-1} > \frac{x-1}{x+2}$$

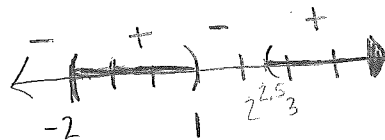
$$\frac{(x+2)}{(x+2)} \cdot \frac{(x-2)}{(x-1)} - \frac{(x-1)}{(x+2)} \cdot \frac{(x-1)}{(x-1)} > 0$$

$$\frac{x^2-4 - (x^2-2x+1)}{(x+2)(x-1)} > 0$$

$$\frac{x^2-4-x^2+2x-1}{(x+2)(x-1)} > 0$$

$$\frac{2x-5}{(x+2)(x-1)} > 0$$

$$(-2, 1) \cup (\frac{5}{2}, \infty)$$



Test -3	Test 0	Test 2	Test 3
$\frac{-}{(-)} = +$	$\frac{-}{(+)} = -$	$\frac{-}{(+)} = -$	$\frac{+}{(+)} = +$

$$10. x+4y=3$$

$$2x-7y=-24$$

$$x=3-4y$$

$$2(3-4y)-7y=-24$$

$$6-8y-7y=-24$$

$$-15y=-30$$

$$y=2$$

$$(5, 2)$$

$$x=3-4(2)$$

$$x=3-8$$

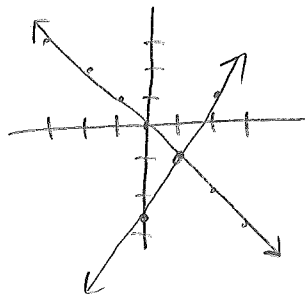
$$x=-5$$

$$11. x+y=0$$

$$y=-x$$

$$3x-2y=5$$

$$y=\frac{3}{2}x-\frac{5}{2}$$



$$(1, -1)$$

$$\begin{array}{r}
 12. \quad 3x - 5y = 8 \\
 + \quad 2x + 5y = 22 \\
 \hline
 5x = 30
 \end{array}$$

$$x = 6$$

$$3(6) - 5y = 8$$

$$\begin{array}{r}
 18 - 5y = 8 \\
 -18 \quad -18
 \end{array}$$

$$-5y = -10$$

$$y = 2$$

$$\boxed{(6, 2)}$$

$$\begin{array}{r}
 14. \quad 4x - 3y - 2z = 21 \\
 \quad \quad 6y - 5z = -8 \\
 \quad \quad \quad z = -2
 \end{array}$$

$$6y - 5(-2) = -8$$

$$6y + 10 = -8$$

$$6y = -18$$

$$y = -3$$

$$4x - 3(-3) - 2(-2) = 21$$

$$4x + 9 + 4 = 21$$

$$4x + 13 = 21$$

$$4x = 8$$

$$x = 2$$

$$\boxed{(2, -3, -2)}$$

$$15. \quad 5x - 3y + 2z = 3$$

$$2 \cdot 2x + 4y - z = 7 \cdot 4$$

$$x - 11y + 4z = 3$$

eliminate z

$$5x - 3y + 2z = 3$$

$$4x + 8y - 2z = 14$$

$$-1 \cdot 9x + 5y = 17$$

$$\rightarrow -9x - 5y = -17$$

$$9x + 5y = 31$$

$$0 = 14$$

$$13. \quad 4x - 3y = 6 \cdot 5$$

$$-5x + 7y = -1 \cdot 4$$

$$20x - 15y = 30$$

$$-20x + 28y = -4$$

$$13y = 26$$

$$y = 2$$

$$4x - 3(2) = 6$$

$$+6 \quad +6$$

$$4x = 12$$

$$x = 3$$

$$\boxed{(3, 2)}$$

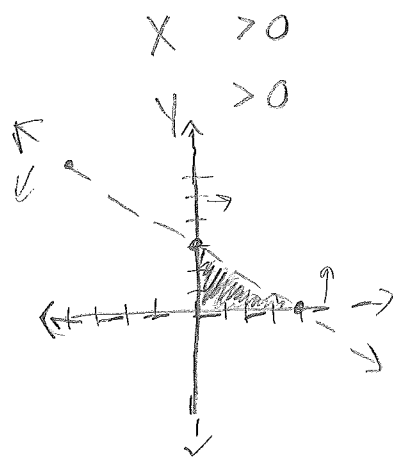
$$8x + 16y - 4z = 28$$

$$x - 11y + 4z = 3$$

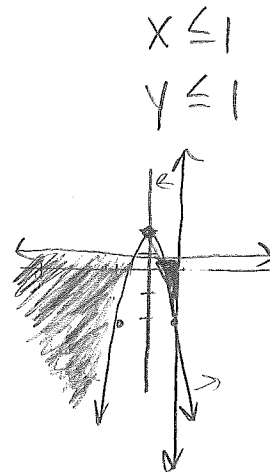
$$9x + 5y = 31$$

No solution
(Inconsistent)

$$16. 3x + 4y < 12 \rightarrow y < -\frac{3}{4}x + 3$$



$$17. 4x^2 + y \geq 2 \rightarrow y \geq -4x^2 + 2$$



$$18. \left[\begin{array}{ccc|c} 4 & -3 & 1 & -2 \\ 0 & 5 & -1 & 6 \\ 2 & 0 & 8 & -9 \end{array} \right]$$

$$19. \begin{array}{cccc} -3 & -6 & -9 & -12 \\ 3 & 3 & 7 & 1 \\ \hline 0 & -3 & -2 & -11 \end{array}$$

$$\left[\begin{array}{ccc|c} 2 & 4 & 5 & -2 \\ 0 & -3 & -2 & -11 \\ 3 & 3 & 7 & 1 \end{array} \right] \text{New } R_2$$

$$20. \left[\begin{array}{cc|c} 2 & -3 & 5 \\ 1 & 4 & -7 \end{array} \right]$$

$$-2 \cdot R_2 + R_1 \rightarrow R_2$$

$$\left[\begin{array}{cc|c} 2 & -3 & 5 \\ 0 & -11 & 19 \end{array} \right]$$

$$-11y = 19$$

$$y = -\frac{19}{11}$$

$$2x - 3\left(-\frac{19}{11}\right) = 5$$

$$2x + \frac{57}{11} = 5$$

$$\frac{1}{2} \cdot 2x = -\frac{2}{11} \cdot \frac{1}{2}$$

$$x = -\frac{1}{11}$$

$$\left(-\frac{1}{11}, -\frac{19}{11} \right)$$

$$21. \left[\begin{array}{ccc|c} 2 & 2 & -1 & 2 \\ 1 & -3 & 1 & -28 \\ -1 & 1 & 0 & 14 \end{array} \right]$$

$$R_1 \leftrightarrow R_2$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & -28 \\ 2 & 2 & -1 & 2 \\ -1 & 1 & 0 & 14 \end{array} \right]$$

$$-2R_1 + R_2 \rightarrow R_2$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & -28 \\ 0 & 8 & -3 & 58 \\ -1 & 1 & 0 & 14 \end{array} \right]$$

$$R_1 + R_3 \rightarrow R_3$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & -28 \\ 0 & 8 & -3 & 58 \\ 0 & -2 & 1 & -14 \end{array} \right]$$

$$4R_3 + R_2 \rightarrow R_3$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & -28 \\ 0 & 8 & -3 & 58 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

$$\frac{1}{8}R_2 \rightarrow R_2$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & -28 \\ 0 & 1 & -\frac{3}{8} & \frac{58}{8} \\ 0 & 0 & 1 & 2 \end{array} \right]$$

$$3R_2 + R_1 \rightarrow R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{8} & -\frac{50}{8} \\ 0 & 1 & -\frac{3}{8} & \frac{58}{8} \\ 0 & 0 & 1 & 2 \end{array} \right]$$

Cont. next page

$$\begin{array}{r} 3 \cdot 58 = 174 \\ \quad 3 \cdot 28 = 84 \\ \hline 174 - 84 = 90 \end{array}$$

21. continued

$$\frac{3}{8}R_3 + R_2 \rightarrow R_2$$

$$\begin{bmatrix} 1 & 0 & \frac{1}{8} & \frac{-50}{8} \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\frac{1}{8}R_3 + R_1 \rightarrow R_1$$

$$\begin{bmatrix} 1 & 0 & 0 & -6 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 2 \end{bmatrix} \quad \begin{array}{l} x = -6 \\ y = 8 \\ z = 2 \end{array}$$

$$\boxed{(-6, 8, 2)}$$

25. $2 \times 3 \cdot 3 \times 2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2+6 & 2+0+12 \\ 0+1+8 & 0+0+16 \end{bmatrix} = \boxed{\begin{bmatrix} 9 & 14 \\ 9 & 16 \end{bmatrix}}$$

$$27. \begin{vmatrix} 3 & 5 \\ -2 & 7 \end{vmatrix} = (3)(7) - (-2)(5) \\ = 21 + 10 \\ = \boxed{31}$$

$$22. \begin{bmatrix} 2+3 & 1+4 \\ 4+2 & -3+7 \end{bmatrix} = \boxed{\begin{bmatrix} -1 & 5 \\ 2 & 4 \end{bmatrix}}$$

$$23. \begin{bmatrix} 6 & 3 \\ 12 & -9 \end{bmatrix} - \begin{bmatrix} -6 & 8 \\ -4 & 14 \end{bmatrix} = \boxed{\begin{bmatrix} 12 & -5 \\ 16 & -23 \end{bmatrix}}$$

$$24. \begin{bmatrix} 4 & 2 \\ 8 & -6 \end{bmatrix} \cdot \begin{bmatrix} -3 & 4 \\ -2 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} -12+4 & 16+14 \\ -24+12 & 32+42 \end{bmatrix} = \boxed{\begin{bmatrix} -16 & 30 \\ -12 & -10 \end{bmatrix}}$$

26. $3 \times 2 \cdot 3 \times 3$

cannot multiply

~~$$28. \begin{vmatrix} -1 & 2 & 1 & -1 & 2 \\ 2 & -2 & 3 & 2 & -2 \\ 3 & -1 & 0 & 3 & -1 \end{vmatrix}$$~~

~~$$= [(-1)(-2)(0) + (2)(3)(3) + (1)(2)(-1)] \\ - [(3)(-2)(1) + (-1)(3)(-1) + (0)(2)(2)] \\ = 16 - -3 \\ = \boxed{19}$$~~

Don't need to do the determinant of a 3x3 by hand.

$$29. \begin{bmatrix} 18 & 12 \\ 30 & 24 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ 23 \end{bmatrix}$$

$$A = \begin{bmatrix} 18 & 12 \\ 30 & 24 \end{bmatrix} \quad B = \begin{bmatrix} 13 \\ 23 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$A \cdot X = B$$

$$X = A^{-1} \cdot B$$

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$|A| = \begin{vmatrix} 18 & 12 \\ 30 & 24 \end{vmatrix} = (18)(24) - (30)(12) \\ = 432 - 360 \\ = 72$$

$$A^{-1} = \frac{1}{72} \begin{bmatrix} 24 & -12 \\ -30 & 18 \end{bmatrix} = \begin{bmatrix} \frac{24}{72} & \frac{-12}{72} \\ \frac{-30}{72} & \frac{18}{72} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & -\frac{1}{6} \\ -\frac{5}{12} & \frac{3}{12} \end{bmatrix}$$

$$A^{-1} \cdot B$$

$$\begin{bmatrix} \frac{1}{3} & -\frac{1}{6} \\ -\frac{5}{12} & \frac{3}{12} \end{bmatrix} \cdot \begin{bmatrix} 13 \\ 23 \end{bmatrix} = \begin{bmatrix} (\frac{1}{3})(13) + (-\frac{1}{6})(23) \\ (-\frac{5}{12})(13) + (\frac{3}{12})(23) \end{bmatrix} = \begin{bmatrix} \frac{13}{3} - \frac{23}{6} \\ -\frac{65}{12} + \frac{69}{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\textcircled{2} \times 2 \quad \textcircled{2} \times 1$$

$$\boxed{\left(\frac{1}{2}, \frac{1}{3}\right)}$$

$$A \cdot X = B$$

$$30. \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ -1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix}$$

$$X = A^{-1} \cdot B$$

$$\begin{bmatrix} 1 & -1 & -1 \\ -3 & 5 & 2 \\ 2 & -3 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix} = \begin{bmatrix} 2+3-4 \\ -6-15+8 \\ 4+9-4 \end{bmatrix} = \begin{bmatrix} 1 \\ -13 \\ 9 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\boxed{(1, -13, 9)}$$

$$31. \boxed{\text{consistent}} \\ \boxed{(-2, 4, 1)}$$

$$32. \boxed{\text{inconsistent}} \\ 0 \neq 1$$

$$33. z = a \\ y + 3a = 4 \quad x = -2 \\ y = 4 - 3a \\ \boxed{\text{consistent}} \\ \boxed{(-2, 4 - 3a, a)}$$

$$34. S_1 = 4(1) - 2 = 2 \\ S_2 = 4(2) - 2 = 6 \\ S_3 = 4(3) - 2 = 10 \\ S_4 = 4(4) - 2 = 14 \\ S_5 = 4(5) - 2 = 18$$

$$\boxed{\{2, 6, 10, 14, 18\}}$$

$$35. a_1 = 2(1)^2 + (1) = 3 \\ a_2 = 2(2)^2 + (2) = 10 \\ a_3 = 2(3)^2 + (3) = 21 \\ a_4 = 2(4)^2 + (4) = 36 \\ a_5 = 2(5)^2 + (5) = 55$$

$$\boxed{\{3, 10, 21, 36, 55\}}$$

$$36. a_1 = 8 \\ a_2 = 5(8) + 2 = 42 \\ a_3 = 5(42) + 2 = 212 \\ a_4 = 5(212) + 2 = 1062$$

$$\boxed{\{8, 42, 212, 1062\}}$$

$$37. a_1 = 3(1) - 1 = 2 \\ a_2 = 3(2) - 1 = 5 \\ a_3 = 3(3) - 1 = 8$$

sum:

$$\boxed{2 + 5 + 8 + 11 + \dots + 3n - 1}$$

$$38. 4^3 + 5^3 + 6^3 + \dots + 13^3$$

$$\boxed{\sum_{i=1}^{10} (i+3)^3}$$

$$\text{or} \\ \sum_{i=4}^{13} i^3$$

$$39. \sum_{i=2}^5 (3i+7)$$

$$\boxed{[3(2)+7] + [3(3)+7] + [3(4)+7] + [3(5)+7]}$$

$$\boxed{13 + 16 + 19 + 22 = 70}$$

$$40. a_1 = 5, d = 3$$

$$a_n = 5 + (n-1)(3)$$

$$a_n = 5 + 3n - 3$$

$$\boxed{a_n = 2 + 3n}$$

$$a_{18} = 2 + 3(18)$$

$$\boxed{a_{18} = 56}$$

$$41. a_1 = 18$$

$$a_2 = 20$$

$$a_3 = 22$$

$$d = 2$$

$$n = 28$$

$$a_n = 18 + (n-1)(2)$$

$$a_n = 18 + 2n - 2$$

$$a_n = 16 + 2n$$

$$a_{28} = 16 + 2(28) = 72$$

$$S_{28} = \frac{28}{2}(18 + 72)$$

$$= 14(90)$$

$$= \boxed{1260 \text{ seats}}$$

$$42. a_7 = -47$$

$$a_{13} = -101$$

$$d = \frac{-101 - (-47)}{13 - 7} = -9$$

$$a_7 = a_1 + (7-1)(-9)$$

$$-47 = a_1 + (-54)$$

$$7 = a_1$$

$$a_n = 7 + (n-1)(-9)$$

$$= 7 - 9n + 9$$

$$= 16 - 9n$$

$$\boxed{a_1 = 7; d = -9; a_n = 16 - 9n}$$

$$46. \sum_{i=1}^5 \left(\frac{1}{2}\right)(2)^i$$

$$a \neq \frac{1}{2}(2)^1 = 1$$

$$r = 2$$

$$S_5 = 1 \left(\frac{1-2^5}{1-2} \right)$$

$$= \frac{1-32}{-1} = \boxed{31}$$

$$47. a) \boxed{\text{diverge}}$$

$$\text{b/c } r = 2$$

$$|2| > 1$$

$$b) \boxed{\text{converge}}$$

$$\text{b/c } r = -\frac{1}{2}$$

$$|-\frac{1}{2}| < 1$$

$$S_{\infty} = \frac{4}{1 - (-\frac{1}{2})} = \frac{4}{\frac{3}{2}} = \boxed{\frac{8}{3}}$$

$$43. (-9) + (-2) + 1 + 4 + \dots + 76$$

$$a_1 = -9, d = 3$$

$$76 = -9 + (n-1)(3)$$

$$76 = -9 + 3n - 3$$

$$84 = 3n$$

$$28 = n$$

$$S_{28} = \frac{28}{2}(-9 + 76)$$

$$= 14(71)$$

$$= \boxed{994}$$

$$44. -1, \frac{1}{2}, -\frac{1}{4}$$

$$r = -\frac{1}{2}$$

$$a_n = -1 \left(-\frac{1}{2}\right)^{n-1}$$

$$a_{10} = -1 \left(-\frac{1}{2}\right)^{10-1}$$

$$= -1 \left(-\frac{1}{2}\right)^9$$

$$= \boxed{\frac{1}{512}}$$

$$45. 2, 6, 18, 54, 162, \dots$$

$$r = \frac{6}{2} = \frac{18}{6} = \text{etc} = 3$$

$$\boxed{a_n = 2(3)^{n-1}}$$

$$48. a) -5n + 2$$

$$\boxed{\text{arithmetic; } d = -5}$$

$$b) 4n^2 + 7$$

$$\boxed{\text{neither}}$$

$$c) 3^n = 9^n$$

$$\boxed{\text{geometric; } r = 9}$$