Math 1050 Final Review

Fall 2016

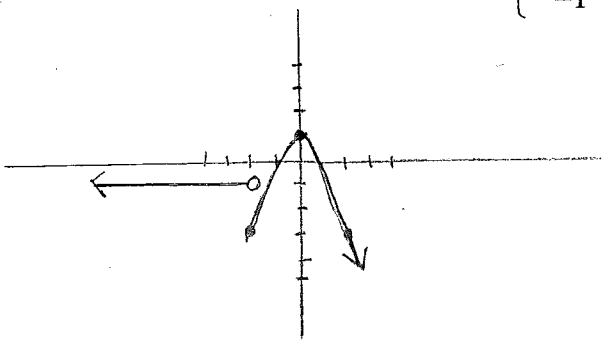
All Functions:

Be able to find the Domain, Range, x-intercept, y-intercept, evaluate points for ALL functions. Also be able to tell if a function is even, odd, or neither; and where a function is increasing, decreasing, and constant. Be able to graph any function.

Piecewise Functions:

Make sure you understand the domain restrictions and which part to plug values into.

1. Given the graph of a piece-wise function $G(x)=\begin{cases} -x^2+1 & x\geq -2 \\ -1 & x<-2 \end{cases}$



Find the domain of G(x). $(-\infty,\infty)$	Find the range of G(x). $(-\infty,1]$
Find G (-2). $-(-2)^2+1$ $-4+1$ $\boxed{-3}$	Determine the interval where the function is increasing. $(-2,0)$
Find G(3). $-(3)^2+1$ $-9+1$ $\boxed{-8}$	Determine the interval where the function is decreasing. $(0,\infty)$
Find G(-3/2). $-\left(-\frac{3}{2}\right)^2+1$ $-\frac{9}{4}+1$ $\boxed{-\frac{5}{4}}$	Determine the interval where the function is constant. $(-\infty,-2)$

Quadratic Functions:

Be able to find the vertex using $x=\frac{-b}{2a}$ to find the *x* coordinate then plug in to find the *y* coordinate, or complete the square. The first way is easier.

Find intercepts by factoring if possible, otherwise use the quadratic formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$.

Find a difference quotient.

2. Given the function, answer the following. $f(x)=2x^2-2x-5$

Find the vertex. (Leave your answer as an ordered pair.) Box your final answer.

$x=\frac{-(-2)}{2(2)}=\frac{2}{4}=\frac{1}{2}$

$f(\frac{1}{2})=2(\frac{1}{2})^2-2(\frac{1}{2})-5$

$=\frac{1}{2}-1-5=-\frac{11}{2}$

$\boxed{(\frac{1}{2},-\frac{11}{2})}$

Find the exact *x*-intercept(s).

$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(2)(-5)}}{2(2)}$

$=\frac{2\pm\sqrt{4+40}}{4}=\frac{2\pm\sqrt{44}}{4}=\frac{2\pm2\sqrt{11}}{4}$

$=\frac{1\pm\sqrt{11}}{2}$

$\boxed{(\frac{1+\sqrt{11}}{2},0),(\frac{1-\sqrt{11}}{2},0)}$

Find the *y*-intercept.

$f(0)=-5$

$\boxed{(0,-5)}$

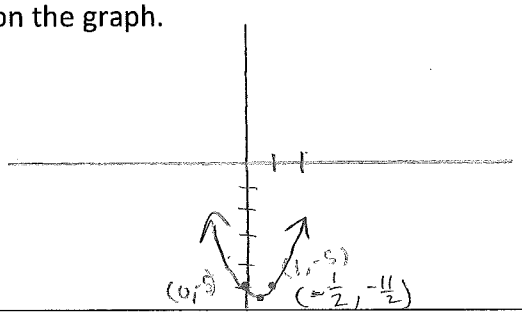
Find the domain of *f(x)*.

$\boxed{(-\infty,\infty)}$

Find the range of *f(x)*.

$\boxed{[-\frac{11}{2},\infty)}$

Graph *f(x)*. Label three points on the graph.



Find the difference quotient of $f(x)=2x^2-2x-5$ where $\frac{f(x+h)-f(x)}{h}$. Simplify and box your answer.

$f(x+h)=2(x+h)^2-2(x+h)-5$

$=2(x^2+2xh+h^2)-2x-2h-5$

$=2x^2+4xh+2h^2-2x-2h-5$

$\frac{(2x^2+4xh+2h^2-2x-2h-5)-(2x^2-2x-5)}{h}$

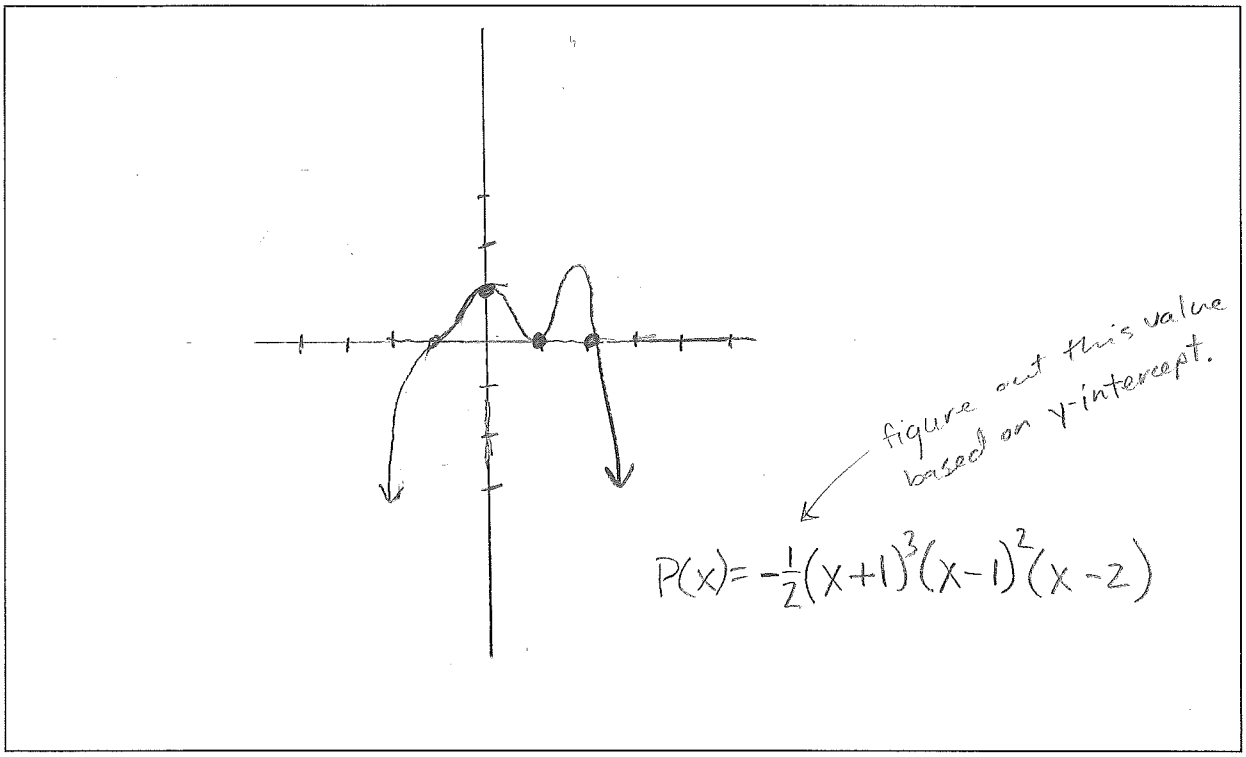
$=\frac{2x^2+4xh+2h^2-2x-2h-5-2x^2+2x+5}{h}$

$=\frac{h(4x+2h-2)}{h}=\boxed{4x-2+2h}$

Polynomial Functions:

Understand multiplicity when looking at a graph, end behavior, intercepts, etc.

3. Below is the graph of a polynomial function *P(x)*. Looking at the properties of the graph (i.e. end behavior, zeros, multiplicity, and intercepts). Write an equation that best describes *P(x)*?



Rational Functions:

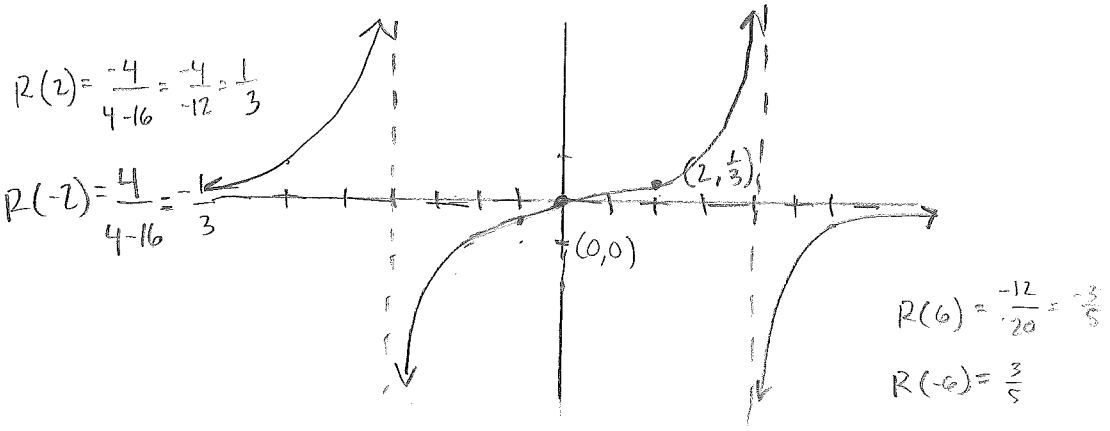
Find vertical and horizontal asymptotes, domain, range, and intercepts.

4. Given the functions, answer the following questions: $R(x)=\frac{-2x}{x^2-16}$

Find the *x*-intercept(s). Top = 0	Find the *y*-intercept.
$-2x=0$ $x=0$ $(0,0)$	$R(0)=\frac{-2(0)}{(0)^2-16}=0$ $(0,0)$
Find the domain of *R(x)*. $x^2-16\neq 0$ $(x+4)(x-4)\neq 0$ $x\neq -4,4$ $(-\infty,-4)\cup(-4,4)\cup(4,\infty)$	

Find any vertical asymptotes. (If not applicable, state so.) Write your answer(s) as an equation.	Find any horizontal asymptotes. (If not applicable, state so.) Write your answer(s) as an equation.
$x=4,\ x=-4$	higher power in denom. $y=0$

Graph *R(x)*. Label three points on the graph. Show all asymptotes.



Is *R(x)* even, odd, or neither? Support your answer with sound algebraic reasoning.

$$R(-x)=\frac{-2(-x)}{(-x)^2-16}=\frac{2x}{x^2-16}$$

odd b/c $R(-x)=-R(x)$

Composition of Functions *f(g(x))* or $(f\circ g)(x)$

5. Given $f(x)=x+1$ and $g(x)=\dfrac{x-5}{x}$, answer the following:

a) Find the composite function $(f\circ g)(x)$.

$$=\frac{x-5}{x}+1\cdot\frac{x}{x}$$

$$=\frac{x-5+x}{x}=\frac{2x-5}{x}$$

b) What is the domain of $(f\circ g)(x)$?

$(-\infty,0)\cup(0,\infty)$

c) Evaluate $(f\circ g)(4)$.

$$=\frac{2(4)-5}{4}=\frac{8-5}{4}=\frac{3}{4}$$

Exponential and Logarithmic Functions:

Be able to change from one to the other, know how to solve each, find the intercepts, asymptotes, domain and range for both.

6. Given the function, answer the following: $f(x) = 2^{x-1}$

Find f^{-1}

$x = 2^{y-1}$

$\log_2 x = y - 1 \rightarrow \log_2 x + 1 = y$

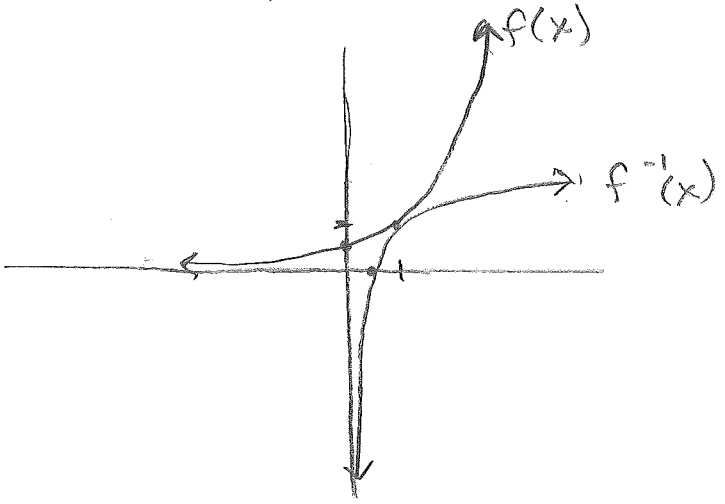
$\boxed{f^{-1}(x) = \log_2 x + 1}$

Find the domain of *f*. $(-\infty, \infty)$	Find the range of *f*. $(0, \infty)$
Find the domain of f^{-1}. $(0, \infty)$	Find the range of f^{-1}. $(-\infty, \infty)$
Find the *y*-intercept of *f*. $f(0) = 2^{0-1} = 2^{-1} = \frac{1}{2}$ $(0, \frac{1}{2})$	Find the *x*-intercept of f^{-1}. $(\frac{1}{2}, 0)$
Find the horizontal asymptote of *f*. $y = 0$	Find the vertical asymptote of f^{-1}. $x = 0$

Graph *f* and f^{-1} on the same coordinate plane. Label both functions.

$f(1) = 1$

$f^{-1}(1) = 1$



7. Solve the equation: $\log x + \log(x+9) = 1$

$\log(x^2+9x) = 1$

$10^1 = x^2+9x$

$x^2+9x-10=0$

$(x+10)(x-1)=0$

$x=-10$, $\boxed{x=1}$

8. The half-life of radium is 1690 years. How long does it take 10 grams of radium to decay to 8 grams? Approximate your final answer to two decimals.

$\frac{1}{2}A_0 = A_0 e^{K\cdot 1690}$

$\frac{1}{2} = e^{1690K}$

$\frac{\ln\frac{1}{2}}{1690} = \frac{1690k}{1690}$

$-4.10146\times10^{-4} = K^*$

$8 = 10e^{K^*\cdot t}$

$\frac{8}{10} = e^{K^*\cdot t}$

$\frac{\ln(\frac{4}{5})}{K^*} = \frac{K^*\cdot t}{K^*}$

$t \approx \boxed{544.06 \text{ years}}$

Conics:

Know which formula belongs to each function. Know how to write an equation given key features.

9. Find the equation of an ellipse where the vertices are at (4,3) and (4,9) and the focus is at (4,8).

(4,9) (4,8) (4,6) (4,3)

$2a = 6$

$a = 3$ $a^2 = 9$ center $(4,6)$

$c = 2$ $c^2 = 4$

$b^2 = 9-4$

$b^2 = 5$

$\boxed{\frac{(x-4)^2}{5} + \frac{(y-6)^2}{9} = 1}$

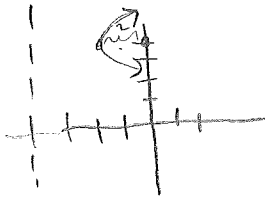
10. Find the vertex, focus, and directrix of the parabola. Graph the equation.

$(y-4)^2 = 8(x+2)$

vertex : $(-2, 4)$

$4p = 8$

$p = 2$



focus : $(0, 4)$

directrix : $x = -4$

11. Find the equation for the hyperbola with center (2, 3), focus (0, 3) and vertex (1, 3).

$$\frac{(x-2)^2}{1} - \frac{(y-3)^2}{3} = 1$$

$c(4,3)$ $v(3,3)$

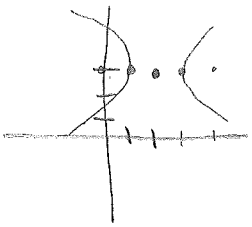
$a = 2-1 = 1$

$c = 2-0 = 2$

$c^2 = a^2 + b^2$

$4 = 1 + b^2$

$3 = b^2$



Matrix Operations:

12. Write the augmented matrix for the system $\begin{cases} 4x-3y+z=-2 \\ 5y-z=6 \\ 2x+8z=-9 \end{cases}$.

$$\left[\begin{array}{ccc|c} 4 & -3 & 1 & -2 \\ 0 & 5 & -1 & 6 \\ 2 & 0 & 8 & -9 \end{array}\right]$$

Know how to solve a system using reduced row echelon form.

13. Solve the system of linear equations given in augmented matrix form:

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 5 \\ 5 & 0 & -3 & 35 \\ 0 & 5 & 4 & 10 \end{array}\right]$$

$-5R1+R2 \to R2$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 5 \\ 0 & 5 & -3 & 10 \\ 0 & 5 & 4 & 10 \end{array}\right]$$

$-R2+R3 \to R3$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 5 \\ 0 & 5 & -3 & 10 \\ 0 & 0 & 7 & 0 \end{array}\right]$$

$\frac{1}{7}R3 \to R3$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 5 \\ 0 & 5 & -3 & 10 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$z = 0$

$5y - 3(0) = 10$

$5y = 10$

$y = 2$

$x - 1(2) + 0(0) = 5$

$x - 2 = 5$

$x = 7$

$\boxed{(7, 2, 0)}$

Solve inequalities:

14. Solve each inequality.

a) $x^3 + 6x^2 + 9x < 0$

$x(x^2 + 6x + 9) < 0$

$x(x+3)^2 < 0$

Key values: $0, -3$

Test -4: $-(+) = -$

Test -1: $(-)(+) = -$

Test 1: $(+)(+) = +$

$\boxed{(-\infty, -3) \cup (-3, 0)}$

b) $\frac{x-2}{x+5} \geq 0$

Key values: $2, -5$ $\quad x \neq -5$

Test -6: $-\div- = +$

Test 0: $-\div+ = -$

Test 3: $+\div+ = +$

$\boxed{(-\infty, -5) \cup [2, \infty)}$

15. Find the sum of the geometric series $2 - \frac{1}{2} + \frac{1}{8} - \frac{1}{32} \ldots$

$r = \frac{-\frac{1}{2}}{2} = -\frac{1}{2} \cdot \frac{1}{2} = -\frac{1}{4}$

$S_\infty = \frac{2}{1 - -\frac{1}{4}} = \frac{2}{\frac{5}{4}} = 2 \cdot \frac{4}{5} = \boxed{\frac{8}{5}}$

16. Find the sum of the finite arithmetic series $\sum_{i=1}^{10} (2i - 1)$

$a_1 = 2(1) - 1 = 1$

$a_{10} = 2(10) - 1 = 19$

$S_{10} = \frac{10}{2}(1 + 19)$

$= 5(20)$

$= \boxed{100}$