

2.3 Measures of Central Tendency**Mean, Median and Mode**

A measure of central tendency is a value that represents a typical, or central entry of a data set. The three most commonly used measures of central tendency are the mean, the median, and the mode.

The mean of a data set is the sum of the data entries divided by the number of entries. To find the mean of a data set, use one of these formulas.

Population Mean:

$$\mu = \frac{\sum x}{N}$$

"mu"

Sample Mean:

$$\bar{x} = \frac{\sum x}{n}$$

"x-bar"

Example 1

The weights (in pounds) for a sample of adults before starting a weight-loss study are listed. What is the mean weight of the adults?

274 235 223 268 290 285 235

$$\sum x = 1810$$

$$\bar{x} = \frac{1810}{7} = \underline{\underline{258.6}} \text{ lbs}$$

round-off rule: go one decimal place more than what is in the data

The heights (in inches) of the players on a professional basketball team are listed. What is the mean height?

74 78 81 87 81 80 77 80 85 78 80 83 75 81 73

$$\sum x = 1193$$

$$\bar{x} = 79.5 \text{ inches}$$

$$\frac{1193}{15}$$

The _____ of a data set is the value that lies in the middle of the data when the data set is ordered. The median measures the center of an ordered data set by dividing it into two equal parts. When the data set has an odd number of entries, the median is the middle data entry. When the data set has an even number of entries, the median is the mean of the two middle data entries.

Example 2

Find the median of the weights and the median of the heights listed in Example 1.

223 235 235 268 274 285 290 268 lbs
 73 74 75 77 78 78 80 80 80 81 81 81 83 85 87 80 in

The ages of a sample of fans at a rock concert are listed. Find the median age.

~~24~~ ~~27~~ ~~19~~ ~~21~~ ~~18~~ ~~23~~ ~~21~~ ~~26~~ ~~19~~ ~~33~~ ~~30~~ ~~29~~ ~~21~~
~~18~~ ~~24~~ ~~26~~ 38 ~~19~~ ~~35~~ ~~34~~ ~~33~~ ~~30~~ ~~21~~ ~~27~~ ~~30~~
 18 18 19 19 19 20 21 21 21 21 23 24 (24) 26 27 27 29 24 yrs
 30 30 30 33 33 34 35

Example 3

In Example 2, the adult weighing 285 pounds decides to not participate in the study. What is the median weight of the remaining adults?

223 235 235 268 274 290 $med = \frac{268+235}{2}$
 $= \frac{503}{2} = \underline{251.5 \text{ lbs}}$

The prices (in dollars) of a sample of digital photo frames are listed. Find the median price of the digital photo frames.

70 ~~10~~ ~~50~~ ~~120~~ ~~80~~ ~~100~~ ~~50~~ ~~120~~ 100 70
 10 50 50 70 70 | 80 100 100 120 130 $med = \frac{70+80}{2} = 75$

The mode of a data set is the data entry that occurs with the greatest frequency. A data set can have one mode, more than one mode, or no mode. When no entry is repeated, the data set has no mode. When two entries occur with the same greatest frequency, each entry is a mode and the data set is called

bimodal.

Example 4

Find the mode of the weights listed in Example 1.

235

The prices (in dollars per square foot) for a sample of South Beach (Miami Beach, FL) condominiums are listed. Find the mode of the prices.

helps to put in order

324 462 540 450 638 564 670 618 624 825
 540 980 1650 1420 670 830 912 750 1260 450
 975 670 1100 980 750 723 705 385 475 720

670

Example 5

At a political debate, a sample of audience members were asked to name the political party to which they belonged. Their responses are shown in the table. What is the mode of the responses?

Political Party	Frequency, f
Democrat	46
Republican	34
Independent	39
Other/don't know	5

46

Democrat

In a survey, 1077 adults ages 18 to 34 were asked why they shop online. Of those surveyed, 312 said "to avoid holiday crowds, hassle," 399 said "better prices," 140 said "better selection," 194 said "convenience," and 32 said "ships directly." What is the mode of the responses?

better prices

An outlier is a data entry that is far removed from the other entries in the data set.

Example 6

Find the mean, the median, and the mode of the sample ages of students in a class. Which measure of central tendency best describes a typical entry of this data? Are there any outliers?

20 20 20 20 20 20 21 21 21 21 22 22 22 23

23 23 23 24 24 65

$$\bar{x} \text{ mean} = \frac{475}{20} \approx 23.8 \text{ years}$$

$$\text{med} = 21.5 \text{ yrs}$$

$$\text{mode} = 20 \text{ yrs}$$

Remove the data entry 65 from the data set in Example 6. Then rework the example. How does the absence of this outlier change each of the measures?

$$\bar{x} = \frac{410}{19} \approx 21.6 \text{ yrs}$$

$$\text{med} = 21$$

$$\text{mode} = 20 \text{ (doesn't affect)}$$

Weighted Mean and Mean of Grouped Data

A weighted mean is the mean of a data set whose entries have varying weights. The weighted mean is given by

$$\bar{x} = \frac{\sum(x \cdot w)}{\sum w}$$

Where w is the weight of each entry x .

Example 7

You are taking a class in which your grade is determined from five sources: 50% from your test mean, 15% from your midterm, 20% from your final exam, 10% from your computer lab work, and 5% from your homework. Your scores are 86 (test mean), 96 (midterm), 82 (final exam), 98 (computer lab), and 100 (homework). What is the weighted mean of your scores? The minimum average for an A is 90. Did you get an A?

No. \bar{x}	Source	Score, x	Weight, w	$x \cdot w$
	Test Mean	86	.50	43
	Midterm	96	.15	14.4
	Final	82 ⁹⁸	.20	16.4 ^{19.6}
	Computer	98	.10	9.8
	Homework	100	.05	5.0
			$\Sigma w = 1$	$\Sigma(x \cdot w) = 88.6$

$\Sigma(x \cdot w) = \frac{91.8}{1}$

88.6

An error was made in grading your final exam. Instead of getting 82, you scored 98. What is your new weighted mean?

The mean of a frequency distribution of a sample is approximated by

where x and f are the midpoint and frequency of each class, respectively.

Guidelines

1. Find the midpoint of each class.
$$x = \frac{\text{lower limit} + \text{upper limit}}{2}$$
2. Find the sum of the products of the midpoints and the frequencies.
$$\Sigma(x \cdot f)$$
3. Find the sum of the frequencies.
$$n = \Sigma f$$
4. Find the mean of the frequency distribution.
$$\bar{x} = \frac{\Sigma(x \cdot f)}{\Sigma f}$$

Example 8

Use the frequency distribution to approximate the mean number of minutes that a sample of Internet subscribers spent online during their most recent session.

Class midpoint, x	Frequency, f	$x \cdot f$
12.5	6	75.0
24.5	10	245.0
36.5	13	474.5
48.5	8	388.0
60.5	5	302.5
72.5	6	435.0
84.5	2	169.0
	$n = 50$	$\Sigma = 2089$

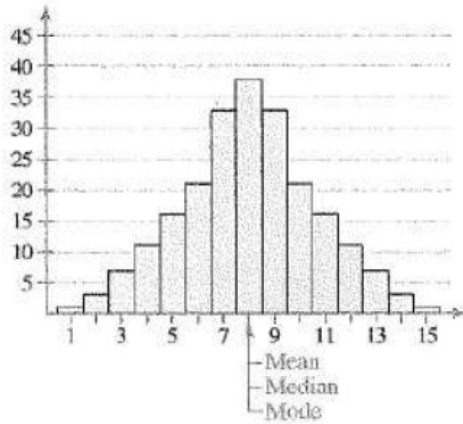
$$\bar{x} = \frac{\Sigma(x \cdot f)}{\Sigma f} = \frac{2089}{50} = 41.8$$

The Shapes of Distributions

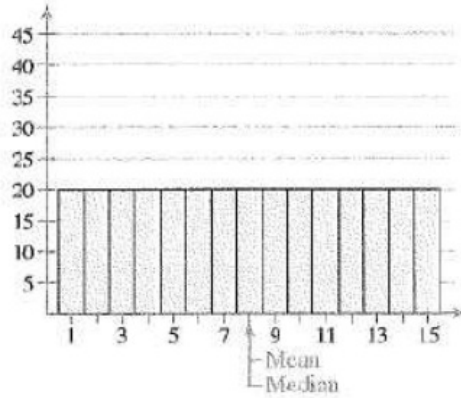
A frequency distribution is symmetric when a vertical line can be drawn through the middle of a graph of the distribution and the resulting halves are approximately mirror images.

A frequency distribution is uniform (or rectangular) when all entries, or classes, in the distribution have equal or approximately equal frequencies. A uniform distribution is also symmetric.

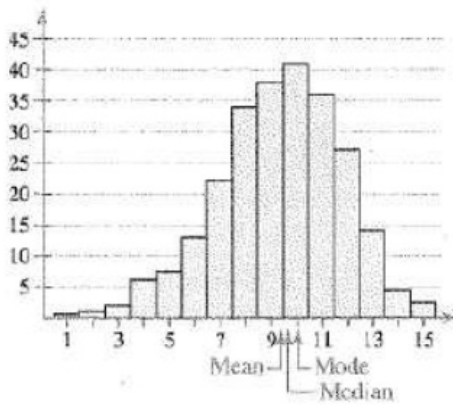
A frequency distribution is skewed when the "tail" of the graph elongates more to one side than to the other. A distribution is skewed left (negatively skewed) when its tail extend to the left. A distribution is skewed right (positively skewed) when its tail extend to the right.



Symmetric Distribution

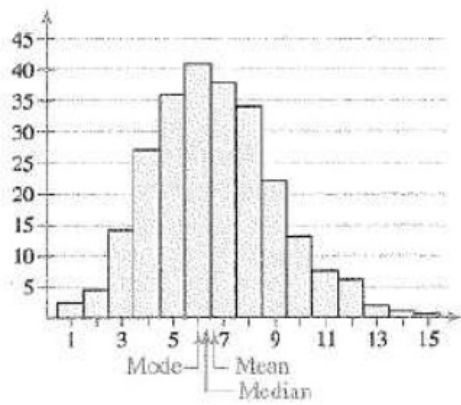


Uniform Distribution



Skewed Left Distribution

mean is left of the median



Skewed Right Distribution

mean is right of the median

the mean is pulled towards the skew