Math 1010 – 2.3 Notes

Linear Functions: Slope, Graphs, and Models

The slope of a line is a measure of its steepness. Steep lines have slopes with higher absolute values than flatter lines.

The slope of the line passing through (x_1, y_1) and (x_2, y_2) is abbreviated m (from the French *monter*, meaning to climb).

$$\text{Slope} = m = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

rise / run $\frac{\Delta y}{\Delta x}$

The slope of the line is the same no matter which two points you use to calculate it.

Positive slopes: Upward from left to right
Negative slopes: Downward from left to right

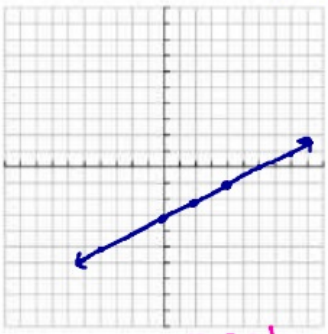
Slope-Intercept Form

Any equation of the form $y = mx + b$ is said to be written in **slope-intercept form**.

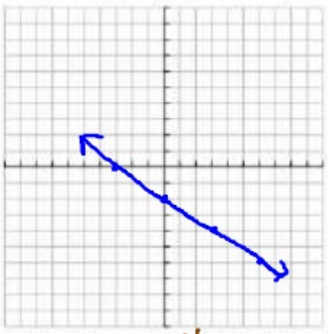
- The graph is a straight line.
- The slope of the line is m.
- The y-intercept of the line is $(0, b)$.

Examples: Determine the slope and the y-intercept; then draw a graph.

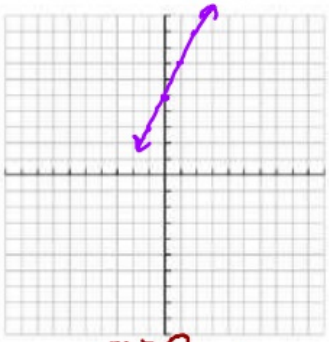
a) $y = \frac{1}{2}x - 3$ m: $\frac{1}{2}$ b: (0,-3)



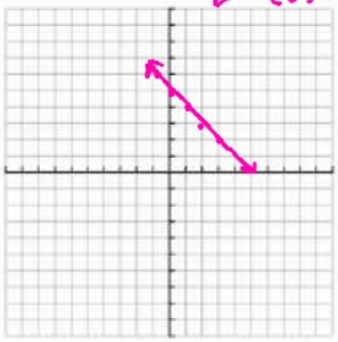
b) $y = -\frac{2}{3}x - 2$ m: $-\frac{2}{3}$ b: (0,-2)



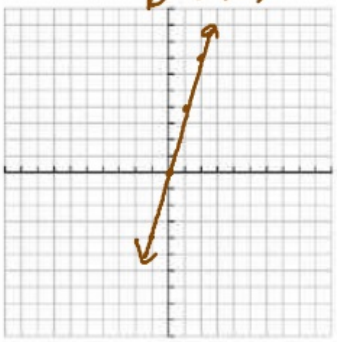
c) $y = 2x + 5$ m: 2 b: (0,5)



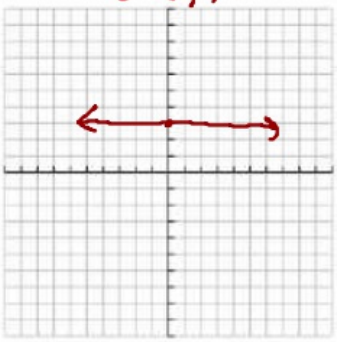
d) $y = -x + 5$ m: -1 b: (0,5)



e) $y = 4x$ m: 4 b: (0,0)



f) $y = 3$ m: 0 b: (0,3)



Examples: For each pair of points, find the slope of the line containing them.

a) $(2,7)$ and $(3,5)$ $\quad \frac{5-7}{3-2} = \frac{-2}{1} = -2$

b) $(-2,4)$ and $(-1,-6)$ $\quad \frac{-6-4}{-1-2} = \frac{-10}{1} = -10$

c) $(8,2)$ and $(4,-4)$ $\quad \frac{-4-2}{4-8} = \frac{-6}{-4} = \frac{3}{2}$

Examples: Find a linear function whose graph has the given slope and y-intercept.

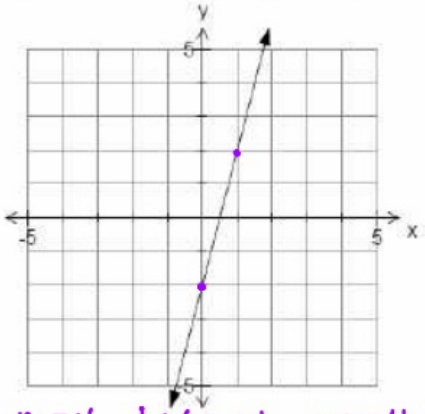
a) Slope: $-\frac{5}{3}$, y-intercept: $(0,2)$ $\quad y = -\frac{5}{3}x + 2$

b) Slope: 2, y-intercept: $(0,-3)$ $\quad y = 2x - 3$

c) Slope: -1, y-intercept: $(0,0)$ $\quad y = -x$

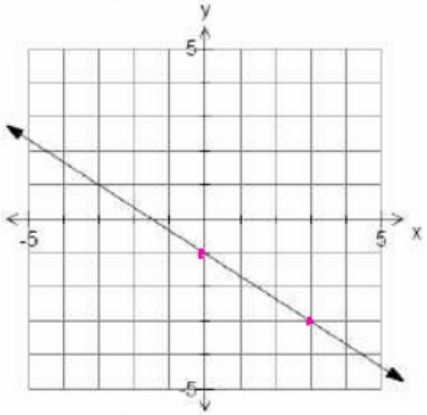
Examples: Find the slope, y-intercept, and an equation for each graph.

a)



$m = 4$ $\quad b: (0,-2)$ $\quad y = 4x - 2$

b)



$m = -\frac{2}{3}$ $\quad b: (0,-1)$ $\quad y = -\frac{2}{3}x - 1$

Applications

In story problems, slope is most often used to represent a *rate of change* (miles per hour, calories per serving of food, etc.)

Examples: Each model is of the form $f(x) = mx + b$. In each, determine what m and b signify.

a) Each salesperson at Super Electronics is paid $P(x)$ dollars, where $P(x) = 0.05x + 200$ and x is the value of the salesperson's sales for the week.

Slope: The salesperson is paid \$0.05 for every \$1.00 in sales.

y-int: The salesperson is paid \$200 as a base salary.

b) After being cut, the length $G(t)$ of the lawn, in inches at Salt Lake Community College is given by $G(t) = \frac{1}{8}t + 2$, where t is the number of days since the lawn was cut.

Slope: The grass grows 1" every 8 days or $\frac{1}{8}$" per day.

y-int: The grass is 2" right after it is cut.

Homework Section 2.3

7,9,12,13,19,23,24,29,33,36,39,42,43,56,61,69,75,79,82,92,95,96