Solving Equations

Expressions vs. Equations: Equations have equal signs, expressions don't. Some of the strategies used to solve equations don't work when you are working with expressions.

Equivalent Expressions: Expressions that have the same value for all possible replacements of each variable.

Working with Expressions: The following operations produce equivalent expressions. (They don't change the value of the expression.)

- Use rules of arithmetic to simplify, including commutative, associative, and distributive properties.
- Combine like terms. (**Terms** are the parts of an expression that are separated by addition or subtractions signs. A term can be a number, variable, or product or quotient of numbers and/or variables. **Like terms** are terms whose variable factors, including exponents, are exactly the same.)
- Use rules of exponents.
- Factor.
- Multiply by 1. (You do this when simplifying fractions and working with square roots.)
- Add 0. (You do this when you complete the square—we'll talk about this later in the year.)

Order of Operations: "Please Excuse My Dear Aunt Sally"

1. Parentheses: Simplify inside parentheses and brackets first. Work inside out.
2. Exponents: Simplify all exponential expressions.
3. Multiplication/Division: Perform multiplication and division from left to right.
4. Addition/Subtraction: Perform addition and subtraction from left to right.

Examples: Simplify to form an equivalent expression by combining like terms. Use the distributive law as needed.

a) $9a-5a^2+4a$

b) $-9n+8n^2+n^3-2n^2-3n+4n^3$

c) $5a-(4a-3)$

d) $5\{-2x+3[2-4(5x+1)]\}$

Equivalent Equations: Two equations are *equivalent* if they have the same solutions.

Solving an Equation: Using the principles of algebra to produce equivalent equations from which the values of the variables are obvious.

The Addition and Multiplication Principles for Equations
For any real numbers a, b, and c:
- $a = b$ is equivalent to $a + c = b + c$.
- $a = b$ is equivalent to $a \cdot c = b \cdot c$, provided $c \neq 0$. (Multiplying by 0 eliminates information.)

There is no need for a subtraction or division principle because subtraction can be regarded as adding opposites and division can be thought of as multiplying by reciprocals.

Golden Rule of Algebra: Whatever you do to one side of an equation, do also to the other.

Types of Equations:
Identity: An equation that is *always* true. The value of the variable can be any real number.
    Solution Set: $\mathbb{R}$ (the set of all real numbers).
Contradiction: An equation that is *never* true. It has no solution.
    Solution Set: $\emptyset$ or $\{\ \}$ (the empty set).
Conditional Equation: An equation that is *sometimes* true. It is true for some values of x and false for other values of x. Most of the equations you are used to dealing with are conditionals.
    Solution Set: The set of x values that make the equation true. e.g.) $\{4\}$ or $\{-5,2\}$.

Homework Section 1.3

1,2,3,4,11,12,17,19,21,23,27,31,37,41,47,51,55,61,63,65,73,77,79,85,86,87,91