Overview of Chapter 7

Antiderivatives

Definite

$\int_1^5 2x\,dx$

$= x^2 \Big|_1^5$

$= 5^2 - 1^2$

$= 24$

Indefinite

$\int 2x\,dx$

$= x^2 + C$

Method for determining antiderivatives (given a function)

- straight forward (some may need to be simplified first)
- substitution
- by parts (after AP test)

Solving *Differential Equations* $\frac{dy}{dx} =$ $y =$

Growth and Decay Problems Exponential Equations

Slope Fields

Section 7.1: Antiderivatives and Differential Equations

Review

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C$$

$$= \frac{x^{n+1}}{n+1} + C$$

Evaluate each indefinite integral

1) $\int 2x^4 dx$ $= 2\int x^4 dx$ $= \frac{2x^5}{5} + C$

2) $\int \frac{1}{x} dx$ $= \ln|x| + C$

3) $\int \frac{2}{3x} dx$ $= \frac{2}{3}\int \frac{1}{x} dx$ $= \frac{2}{3}\ln|x| + C$

4) $\int (-x^{-3}+x-1)dx$

$= \frac{1}{2}x^{-2}+\frac{1}{2}x^{2}-x+C$

or $\frac{1}{2x^2}+\frac{x^2}{2}-x+C$

5) $\int \frac{2}{\sqrt[3]{x}}dx$

$= \int 2x^{-\frac{1}{3}}dx$

$= \frac{2}{\frac{2}{3}}x^{2/3}+C$

$= 2\cdot\frac{3}{2}x^{2/3}+C$

$= 3x^{2/3}+C$

Review

$\frac{d}{dx}(\sin(5x)) = 5\cos(5x)$

6) $\int 3\cos(3x)dx$

$= \sin(3x)+C$

7) $\int \sin(5x)dx$

$= -\frac{1}{5}\cos(5x)+C$

8) $\int \sec(2t)\tan(2t)dt$

$= \frac{1}{2}\sec(2t)+C$

9) $\int -3\csc x\cot x dx$

$= 3\csc x + C$

$\int dy = \int(3x^2-\csc^2 x)dx$

10) $\frac{dy}{dx} = 3x^2-\csc^2 x$

$y = x^3+\cot x+C$

$\int dy = \int(4t^3\sec^2(t^4))dt$

11) $\frac{dy}{dt} = 4t^3\sec^2(t^4)$

$y = \tan(t^4)+C$

Differentiable Equations

Solve the *initial value* problem

1) $\frac{dy}{dx} = 9x^2 - 4x + 5 \quad y(-1) = 0$

$\int dy = \int (9x^2 - 4x + 5)\,dx$

$y = 3x^3 - 2x^2 + 5x + C$

plug in the initial value to find C

$(-1, 0)$

$0 = 3(-1)^3 - 2(-1)^2 + 5(-1) + C$

$0 = -3 - 2 - 5 + C$

$C = 10$

$y = 3x^3 - 2x^2 + 5x + 10$

2) $\frac{dy}{dx} = \frac{1}{x^2} + x$ and $y = 1$ when $x = 2$

$\int dy = \int \left(\frac{1}{x^2} + x\right) dx$

$y = -\frac{1}{x} + \frac{1}{2}x^2 + C$

$y(2) = 1$

$1 = -\frac{1}{2} + \frac{1}{2}(2)^2 + C$

$1 = \frac{3}{2} + C$

$-\frac{3}{2} \quad -\frac{3}{2}$

$-\frac{1}{2} = C$

$y = -\frac{1}{x} + \frac{1}{2}x^2 - \frac{1}{2}$

3) $\frac{d^2y}{dx^2} = 2 - 6x$ $y(0) = 1$ $y'(0) = 4$

$\int d^2y = \int (2-6x)\,dx^2$

$dy = (2x - 3x^2)\,dx + C$

$\frac{dy}{dx} = 2x - 3x^2 + C$

$y'(0) = 4$

$4 = 2(0) - 3(0)^2 + C$

$C = 4$

$\int dy = \int (2x - 3x^2 + 4)\,dx$

$y = x^2 - x^3 + 4x + C$

$1 = (0)^2 - (0)^3 + 4(0) + C$

$C = 1$

$y = x^2 - x^3 + 4x + 1$

$dx \cdot \frac{dy}{dx} = (2x - 3x^2 + 4)\,dx$

4) Solve the initial value problem using the Fundemental Theorem.
(Hint: your answer will contain a definite integral)

$\frac{dy}{dx} = \sqrt[4]{\tan x}$ and $y = 6$ when $x = 3$

$dy = (\sqrt[4]{\tan x})\,dx$

$y = \int_3^x \sqrt[4]{\tan t}\,dt + 6$