Implicit Differentiation

Section 4.2

$\frac{d}{dx}(y) = \frac{dy}{dx}$

$\frac{d}{dx}(y^5) = 5y^4\frac{dy}{dx}$

$\frac{d}{dx}(x) = \frac{dx}{dx} = 1$

$\frac{d}{dx}(3x) = 3\frac{dx}{dx} = 3$

$\frac{d}{dx}(4x^7) = 28x^6\frac{dx}{dx}$

$28x^6$

Find $\frac{dy}{dx}$

1) $y = x^2 - 3x + 2$

$\frac{dy}{dx} = 2x - 3$

2) $5 = 2x + 3y - x^2$

$0 = 2 + 3\frac{dy}{dx} - 2x$

$-3\frac{dy}{dx} = -2x + 2$

$\frac{dy}{dx} = \frac{2}{3}x - \frac{2}{3}$

3) $4x = 7x^2 - \frac{3x^5}{5} + 9y^3$

$4 = 14x - 3x^4 + 27y^2\frac{dy}{dx}$

$3x^4 - 14x + 4 = 27y^2\frac{dy}{dx}$

$\frac{3x^4 - 14x + 4}{27y^2} = \frac{dy}{dx}$

Review $y = \cos(2x)$

$\frac{dy}{dx} = -\sin(2x)\cdot 2$

4) $x + \cos y = 5$

$1 - \sin y\frac{dy}{dx} = 0$

$-\sin y\frac{dy}{dx} = -1$

$\frac{dy}{dx} = \frac{1}{\sin y}$

$\frac{dy}{dx} = \csc y$

5) $\tan x - x^2 \cdot y = 4$

$\sec^2 x - \left(x^2\frac{dy}{dx} + y\cdot 2x\right) = 0$

$\sec^2 x - x^2\frac{dy}{dx} - 2xy = 0$

$-x^2\frac{dy}{dx} = 2xy - \sec^2 x$

$\frac{dy}{dx} = \frac{2xy - \sec^2 x}{-x^2}$

6) $5xy - 3x^2y = \pi^4$

$5x\frac{dy}{dx} + 5y - \left(3x^2\frac{dy}{dx} + 6xy\right) = 0$

$5x\frac{dy}{dx} + 5y - 3x^2\frac{dy}{dx} - 6xy = 0$

$\left(5x - 3x^2\right)\frac{dy}{dx} = 6xy - 5y$

$\frac{dy}{dx} = \frac{6xy - 5y}{5x - 3x^2}$

7) $3x - 2xy + \sin y = 5$

$3 - \left(2x\frac{dy}{dx} + 2y\right) + \cos y\frac{dy}{dx} = 0$

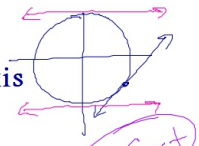
$3 - 2x\frac{dy}{dx} - 2y + \cos y\frac{dy}{dx} = 0$

$\left(-2x + \cos y\right)\frac{dy}{dx} = 2y - 3$

$\frac{dy}{dx} = \frac{2y - 3}{-2x + \cos y}$

8) $x^2 + y^2 = 25$ a) Find the tangent line at (3,-4)

b) Find the tangent line parallel to the x-axis



a)

point (3,-4) slope $\frac{3}{4}$ $y+4=\frac{3}{4}(x-3)$

$2x+2y\frac{dy}{dx}=0$

$2(3)+2(-4)\frac{dy}{dx}=0$

$6-8\frac{dy}{dx}=0$

$6=8\frac{dy}{dx}$

$\frac{dy}{dx}=\frac{3}{4}$

b) point $(0, \quad)$

$0^2+y^2=25$

$y^2=25$

$y=\pm 5$

$(0,5)$

$(0,-5)$

slope $m=0$ Do first

$2x+2y\frac{dy}{dx}=0$

$2x+2y(0)=0$

$2x=0$

$x=0$

$y-5=0(x-0) \Rightarrow y=5$

$y+5=0(x-0) \Rightarrow y=-5$

9) Find the tangent to the curve $x^2 - xy + y^2 = 7$ at (-1,2)

point (-1,2) slope $m=\frac{4}{5}$

$2x-(x\frac{dy}{dx}+y\cdot 1)+2y\frac{dy}{dx}=0$

$2(-1)-(-1)\frac{dy}{dx}-(2)+2(2)\frac{dy}{dx}=0$

$-2+\frac{dy}{dx}-2+4\frac{dy}{dx}=0$

$-4+5\frac{dy}{dx}=0$

$5\frac{dy}{dx}=4$

$\frac{dy}{dx}=\frac{4}{5}$

$y-2=\frac{4}{5}(x+1)$

Review:

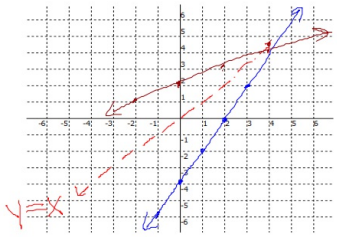
Find the inverse of: $f(x)=2x-4$

$x=2y-4$

$x+4=2y$

$\frac{x+4}{2}=y$

$f^{-1}(x)=\frac{1}{2}x+2$



Simplify: $\frac{d}{dx}(f[g(x)]) = f'(g(x))\cdot g'(x)$

What can you say about the composition of a function and it's inverse?

$h(h^{-1}(x)) = x \qquad h^{-1}(h(x)) = x$

10) Find the slope of the inverse of any function $\frac{d}{dx}[h^{-1}(x)]$ *or* $[h^{-1}]'$

Given: $h(h^{-1}(x)) = x$

$\frac{d}{dx}[h(h^{-1}(x))] = \frac{d}{dx}x$

$h'(h^{-1}(x))\cdot\frac{d}{dx}(h^{-1}(x)) = 1$

$\frac{d}{dx}(h^{-1}(x)) = \frac{1}{h'(h^{-1}(x))}$