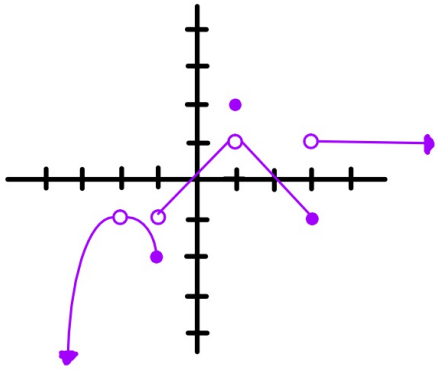
Limits and Rates of Change
Section 2.1

Limits at a point:



$\lim_{x\to 1^-} f(x) = 1$

$\lim_{x\to 1^+} f(x) = 1$

$\lim_{x\to 1} f(x) = 1$

$f(1) = 2$

$\lim_{x\to -2^-} f(x) = -1$

$\lim_{x\to -2^+} f(x) = -1$

$\lim_{x\to -2} f(x) = -1$

$f(-2) = $ d.n.e

does not exist

$\lim_{x\to -1^-} f(x) = -2$

$\lim_{x\to -1^+} f(x) = -1$

$\lim_{x\to -1} f(x) = $ d.n.e.

$f(-1) = -2$

$\lim_{x\to 3^-} f(x) = -1$

$\lim_{x\to 3^+} f(x) = 1$

$\lim_{x\to 3} f(x) = $ d.n.e.

$f(3) = -1$

Algebraically:

- plug in the number
- factor and simplify then plug in the number

1. $\lim_{x\to 3}(x+2) =$

$3+2$

$= \boxed{5}$

2. $\lim_{x\to -3}(x^2 - x - 6) =$

$(-3)^2 - (-3) - 6$

$9 + 3 - 6$

$= \boxed{6}$

3. $\lim_{x\to 2} \frac{x}{x^2-4} =$

$\frac{2}{(2)^2-4} = \frac{2}{4-4}$ = undefined

$\boxed{\text{d.n.e}}$

4. $\lim_{x\to 3} \frac{x^2-9}{x-3} = \frac{3^2-9}{3-3} = \frac{0}{0}$ Need to factor

$= \lim_{x\to 3} \frac{(x+3)\cancel{(x-3)}}{\cancel{x-3}}$

$= \lim_{x\to 3} x+3$

$= 3+3$

$= \boxed{6}$

5. $\lim_{x\to 2} \frac{x-2}{x^2-4} =$

$= \lim_{x\to 2} \frac{\cancel{x-2}\ 1}{(x+2)\cancel{(x-2)}\ 1}$

$= \lim_{x\to 2} \frac{1}{x+2}$

$= \frac{1}{2+2}$

$= \boxed{\frac{1}{4}}$

Memorize

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

6. $\lim_{x\to 0} \frac{\sin(2x)}{x} =$

$$= \lim_{x\to 0} \frac{2\sin x \cos x}{x}$$

$$= \lim_{x\to 0} 2 \cdot \frac{\sin x}{x} \cdot \cos x$$

$$= 2 \cdot 1 \cdot \cos(0)$$

$$= \boxed{2}$$

7. $\lim_{x\to 0} \frac{\sin x + x}{x} =$

$$= \lim_{x\to 0} \frac{\sin x}{x} + \frac{x}{x}$$

$$= \lim_{x\to 0} \frac{\sin x}{x} + 1$$

$$= 1 + 1$$

$$= \boxed{2}$$

8. $\lim_{x\to \pi} \frac{\sin x + x}{x} =$

$$= \frac{\sin \pi + \pi}{\pi}$$

$$= \frac{0 + \pi}{\pi}$$

$$= \boxed{1}$$

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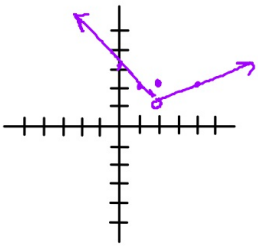
$$c = 2 \qquad f(x) = \begin{cases} 3 - x, & x < 2 \\ 2, & x = 2 \\ \frac{x}{2} & x > 2 \end{cases}$$

a)



b) $\lim_{x\to c^+} f(x) = 1$

$\lim_{x\to c^-} f(x) = 1$

c) Does $\lim_{x\to c} f(x)$ exist?

yes

$$\lim_{x\to c^+} f(x) = \lim_{x\to c^-} f(x)$$