Normal Distributions, part a

Section 2.2

Normal Distributions – The Basics

- Density curves (total area = 1)
- Symmetric
- Single-peaked
- Bell-shaped
- All have the same shape
- "Normal" does not mean average or natural in this case

Normal Distributions - Examples

Normal Distributions

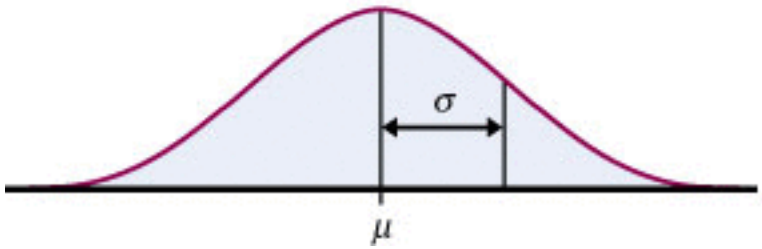
- Good approximations of chance outcomes…like flipping a coin many times.
- Many statistical inference procedures based on Normal distributions work well for other roughly symmetric distributions.
- Although many data sets follow a Normal distribution, many do not.
- "Non-normal data, like non-normal people, not only are common but are sometimes more interesting than their Normal counterparts." ha ha ha

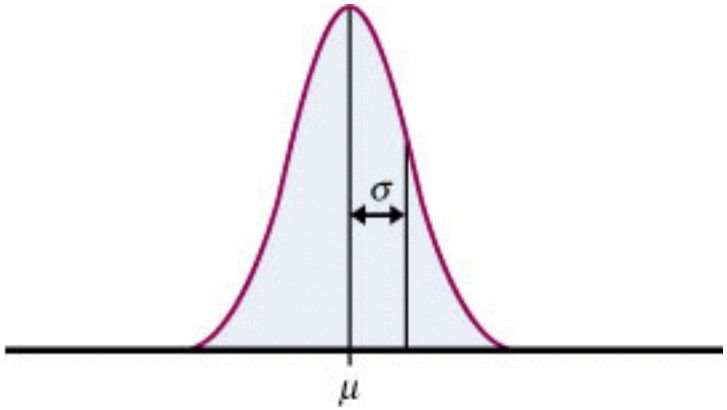
Normal Distributions

- Described by μ (mean) and σ (standard deviation). We abbreviate them as $N(\mu, \sigma)$.
- Shifting μ shifts the center of the distribution.
- Shifting σ shifts the spread of the distribution.

Normal Distributions

- You can find σ by finding the inflexion point of the curve.
- This only works for Normal distributions.
- Generally, you cannot tell anything about the shape of a distribution with μ and σ.

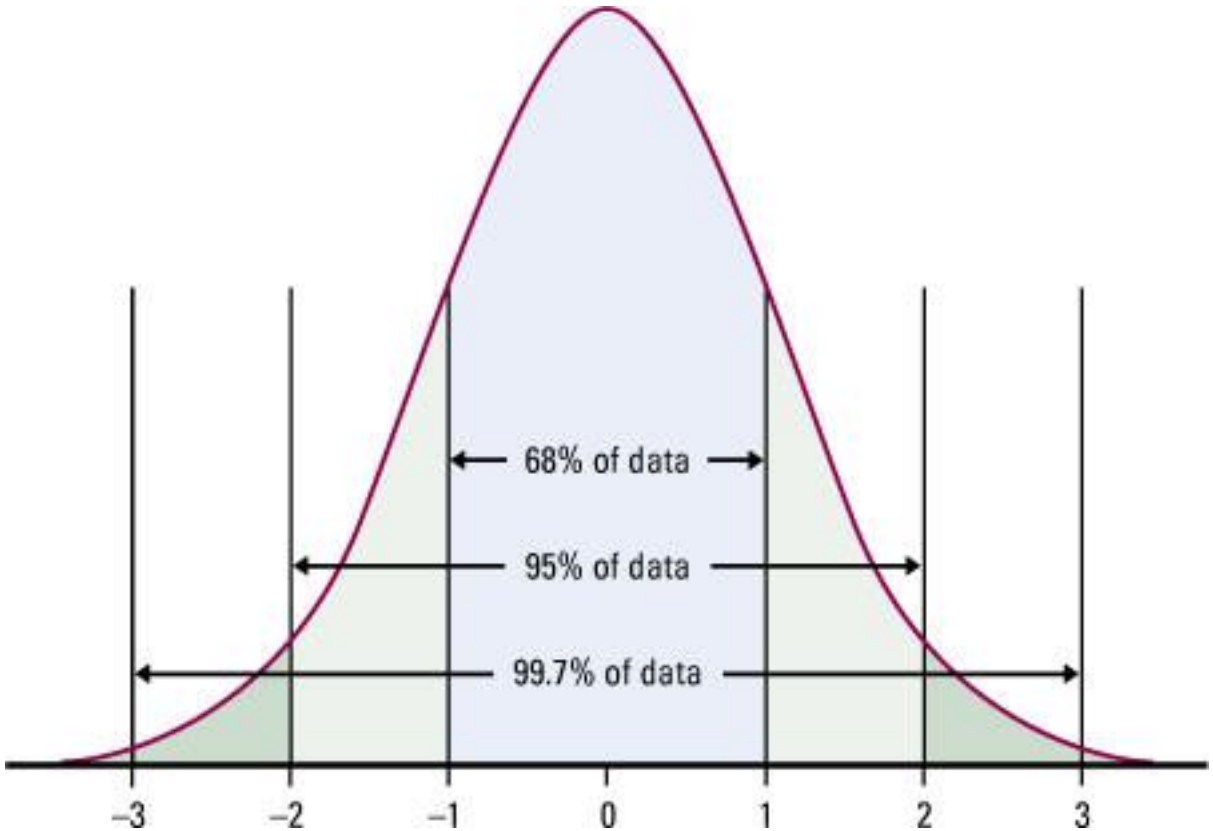




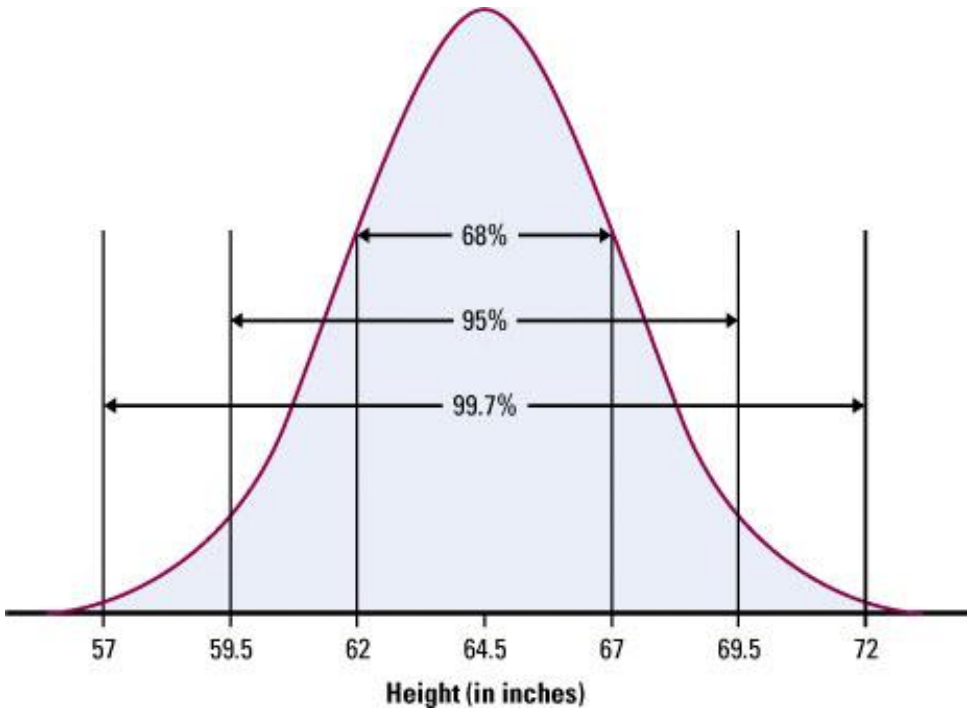
68-95-99.7 Rule

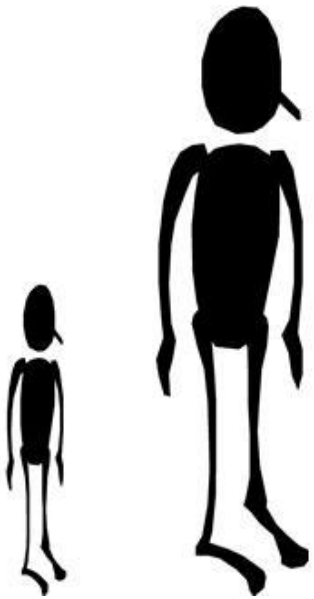
- In the Normal distribution with mean μ and standard deviation σ:
  - Approximately **68%** of the observations fall within σ of the mean μ.
  - Approximately **95%** of the observations fall within 2σ of μ.
  - Approximately **99.7%** of the observations fall within 3σ of μ.

68-95-99.7 Rule



Example 2.6 – Young Women's Heights





$N(64.5, 2.5)$

applet

Calculus to find areas? Nah.

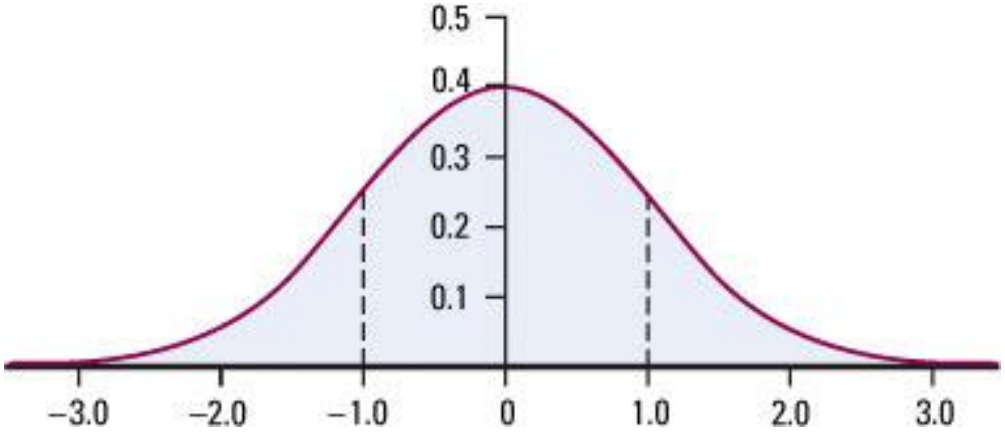
- How do we find these areas if we don't know the function and the calculus required to do so?
- Well, we convert our values to z-scores and use what is called a Standard Normal Distribution.

The Standard Normal Distribution

- A distribution of z-scores of data will be Normal if the original data followed a Normal distribution.
- This new distribution is called the Standard Normal Distribution and has a mean of 0 and standard deviation of 1.
- $N(0,1)$

$$z = \frac{x - n}{s}$$

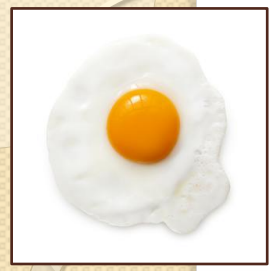
The Standard Normal Distribution



- This model can be used for any data with a Normal distribution, once that data is standardized.
- With *this* model, we **can** directly convert z-scores to percentiles…using a Standard Normal Table.
- Try it…
- Be careful about what is being asked ("to the left" or "to the right")

Solving Problems Involving Normal Distributions

1. State the problem in original terms and sketch a picture.
2. Standardize and sketch a new picture.
3. Use the table.
4. Write your conclusion.

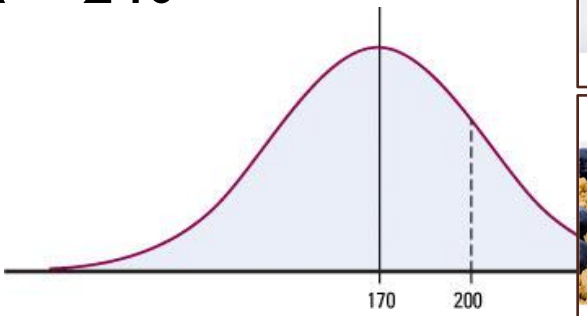


Example 2.8 – Got Probs With Cholesterol



1. State the problem in original terms…

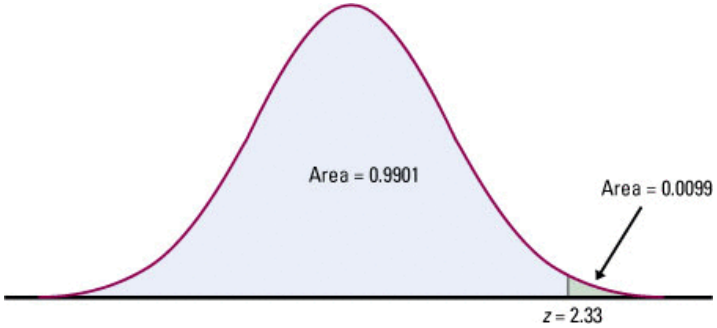
   $N(170, 30)$, looking for $x > 240$



1. Standardize…

   $N(0,1)$, looking for $z > 2.33$

http://muktu.hubpages.com/hub/high-cholesterol-foods-to-avoid



3. Use the table…
4. Conclusion – Only about 1% of 14-year-old boys have dangerously high cholesterol.

In Summary…

- We can standardize data sets (we especially like standardizing Normal data sets).
- Once standardized, we can use the Standard Normal Distribution and a Table to approximate all kinds of cool and crazy things about individual values and the sample as a whole.