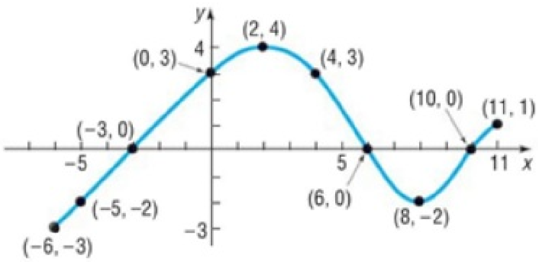


a) Find $f(0)$ and $f(-6)$.

3 -3

b) Find $f(6)$ and $f(11)$.

0 1

c) Is $f(3)$ positive or negative?

d) Is $f(-4)$ positive or negative?

e) For what values of x is $f(x)=0$?

y=0 when x is?

-3, 6, 10

f) For what values of x is $f(x)>0$?

$(-3,6) \cup (10,11]$

g) What is the domain of f?

$[-6, 11]$

h) What is the range of f?

$[-3, 4]$

i) What are the x-intercepts? (-3,0), (6,0), (10,0)

j) What is the y-intercept? (0,3)

k) How often does the line $y=1/2$ intersect the graph?

3 times

l) How often does the line $x=5$ intersect the graph?

once

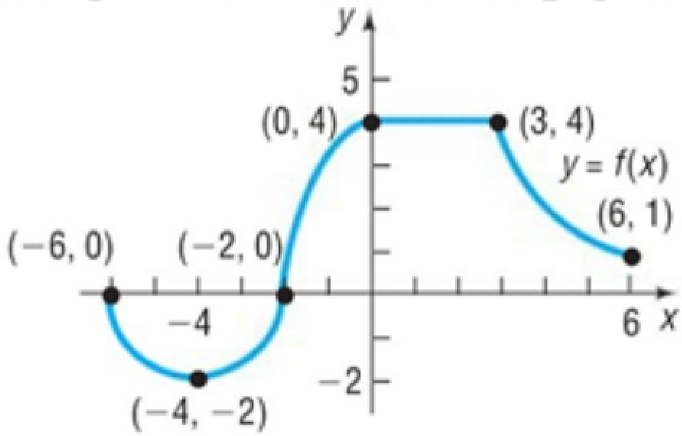
m) For what values of x does $f(x)=3$?

y=3 when x is? 0, 4

n) For what values of x does $f(x)=-2$?

y=-2 when x is? -5, 8

Example: Determine where the graph is increasing, decreasing, or constant.



increasing: $(-4,0)$

decreasing: $(-6,-4) \cup (3,6)$

constant: $(0,3)$

Graphing Piecewise-Defined Functions

Sometimes a function is defined differently on different parts of its domain. When functions are defined by more than one equation, they are called ***piecewise-defined functions.***

Examples: For the following functions:

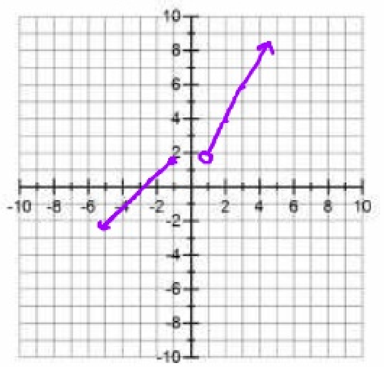
a) Graph the function.

b) Find the domain and range of the function.

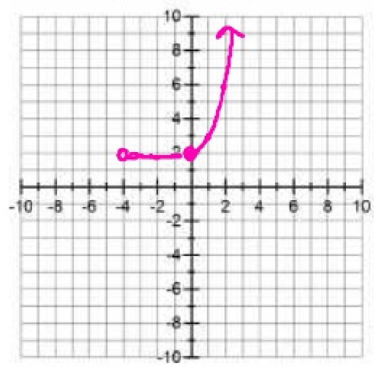
c) Locate any intercepts.

d) State whether the function is continuous on its domain.

1) $f(x)=\begin{cases} x+3 & \text{if } x\le -1 \\ 2x & \text{if } x>1 \end{cases}$

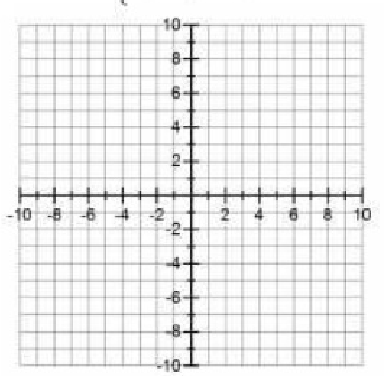


2) $f(x)=\begin{cases} 2 & \text{if } -4<x<0 \\ x^2+2 & \text{if } x\ge 0 \end{cases}$

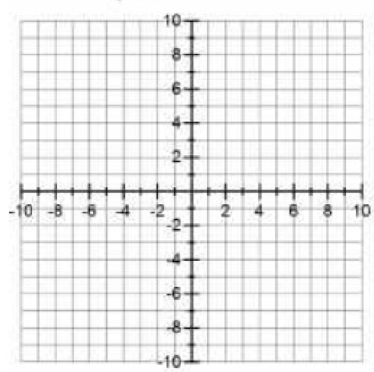


x	x^2+2
0	$0^2+2=2$
1	$1^2+2=3$
2	$2^2+2=6$

3) $f(x)=\begin{cases} 3-x & \text{if } -5\le x<-2 \\ \sqrt{x} & \text{if } 0<x<4 \\ 2x-6 & \text{if } x\ge 4 \end{cases}$



4) $f(x)=\begin{cases} |x| & \text{if } x<2 \\ 5 & \text{if } x=2 \\ -\frac{1}{2}x & \text{if } x>2 \end{cases}$



Graphing Techniques: Transformations

Parent Graph: $y = f(x)$

Offspring: Transformations of the parent graph.

	$f(x)=x^2$	$f(x)=\sqrt{x}$	$f(x)=\frac{1}{x}$	Effect on Parent Graph
$y=f(x)+2$	$f(x)=x^2+2$	$f(x)=\sqrt{x}+2$	$f(x)=\frac{1}{x}+2$	up 2
$y=f(x)-2$	$f(x)=x^2-2$	$f(x)=\sqrt{x}-2$	$f(x)=\frac{1}{x}-2$	down 2
$y=f(x+2)$	$f(x)=(x+2)^2$	$f(x)=\sqrt{x+2}$	$f(x)=\frac{1}{x+2}$	left 2
$y=f(x-2)$	$f(x)=(x-2)^2$	$f(x)=\sqrt{x-2}$	$f(x)=\frac{1}{x-2}$	right 2
$y=2f(x)$	$f(x)=2x^2$	$f(x)=2\sqrt{x}$	$f(x)=\frac{2}{x}$	vertical stretch (narrow)
$y=\frac{1}{2}f(x)$	$f(x)=\frac{1}{2}x^2$	$f(x)=\frac{1}{2}\sqrt{x}$	$f(x)=\frac{1}{2x}$	vertical shrink (wide)
$y=f(2x)$	$f(x)=(2x)^2$	$f(x)=\sqrt{2x}$	$f(x)=\frac{1}{2x}$	horizontal shrink (narrow)
$y=f(\frac{1}{2}x)$	$f(x)=(\frac{1}{2}x)^2$	$f(x)=\sqrt{\frac{1}{2}x}$	$f(x)=\frac{2}{x}$	horizontal stretch
$y=-f(x)$	$f(x)=-x^2$	$f(x)=-\sqrt{x}$	$f(x)=\frac{-1}{x}$	reflect over x-axis
$y=f(-x)$	$f(x)=(-x)^2$	$f(x)=\sqrt{-x}$	$f(x)=\frac{1}{-x}$	reflect over y-axis

When graphing a transformed graph based on an equation, apply transformations in the following order:

1. Stretch/Shrink
2. Reflect
3. Shift, Shift

STRESS

Examples: List the transformations in the appropriate order:

Parent graph: $y=\sqrt{x}$

a) $y=-\frac{1}{2}\sqrt{x+3}$

v. shrink by $\frac{1}{2}$
reflect over x-axis
left 3

b) $y=5\sqrt{-4x}+3$

v. stretch by 5
h. shrink by 4
reflect over y-axis
up 3

c) $y=3\sqrt{-2x+9}$

v. stretch by 3
h. shrink by -2
reflect over y-axis
right 9

Parent graph: $f(x)=|x|$

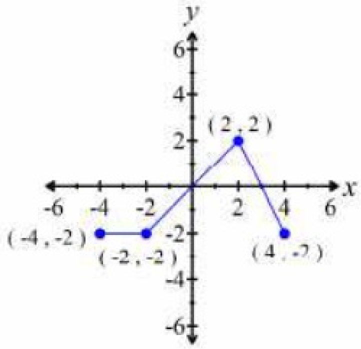
a) $f(x)=4|x-2|+7$

b) $f(x)=-|x+5|-3$

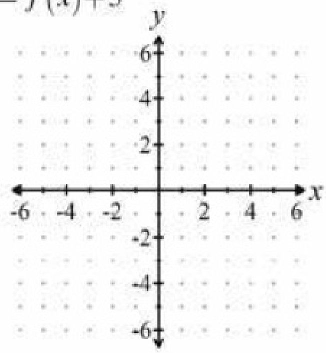
c) $f(x)=-\left|\frac{x}{3}+2\right|$ $\frac{1}{3}x$

h. stretch by $\frac{1}{3}$
reflect over x-axis
left 2

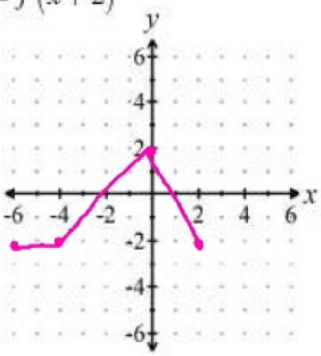
Example: The graph of a function f is illustrated below. Use the graph of f as the first step towards graphing each of the following functions:



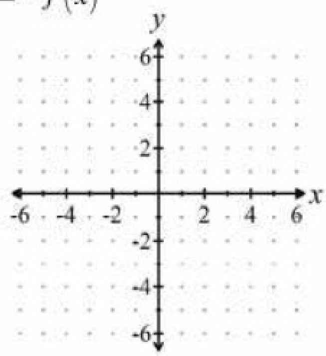
a) $F(x) = f(x) + 3$



b) $G(x) = f(x+2)$

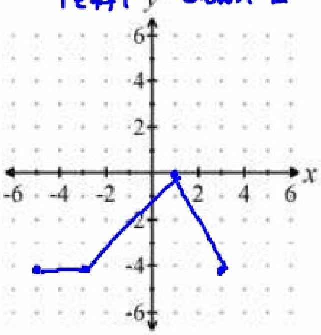


c) $P(x) = -f(x)$

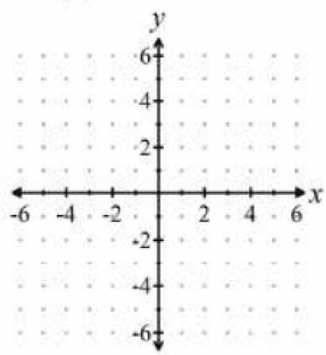


d) $H(x) = f(x+1) - 2$

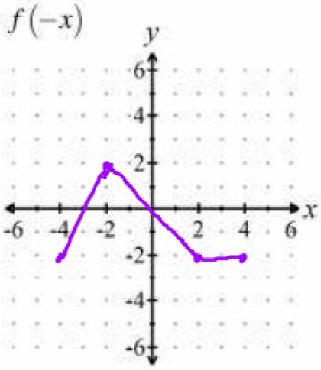
left 1 down 2



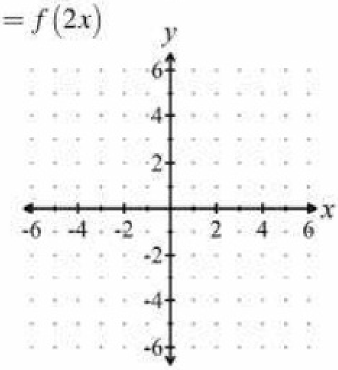
e) $Q(x) = 2f(x)$



f) $g(x) = f(-x)$

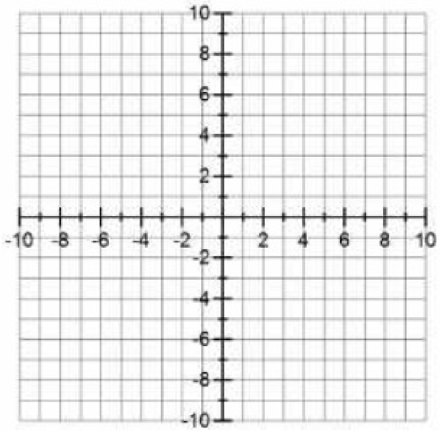


g) $h(x) = f(2x)$



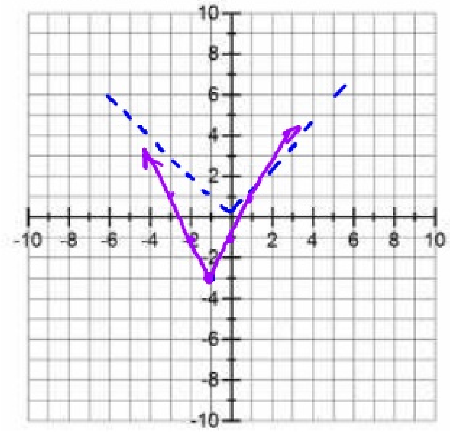
Examples: Graph the following:

a) $f(x)=(x-1)^3+2$

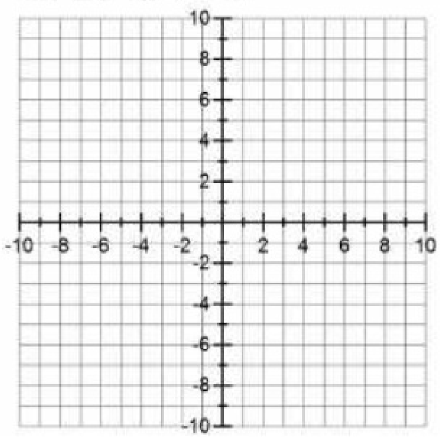


b) $g(x)=2|x+1|-3$

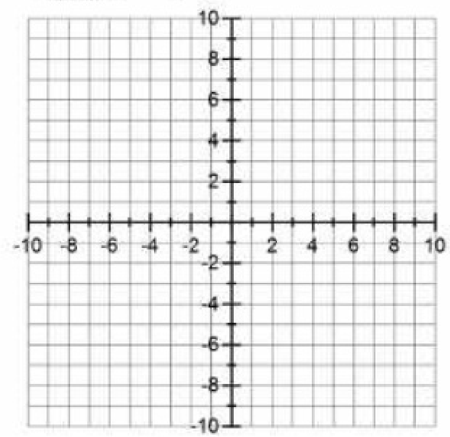
v. stretch by 2
left 1
down 3



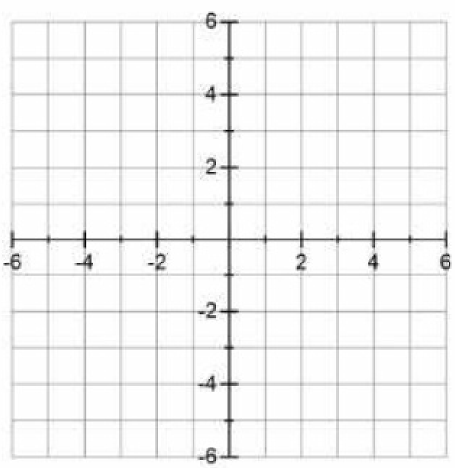
c) $f(x)=\sqrt{-(x-3)}+2$



d) $g(x)=-\sqrt[3]{2x}$



e) $h(x)=\dfrac{3}{(x+2)}$



(Use function notation to write in terms of f.

Example: Write the equation of the function that is graphed after the following transformations are applied in order to the graph of $g(x)=x^3$.

1. Shift down 4 units
2. Reflect across y-axis
3. Vertical compression by a factor of 1/2

$f(x)=\frac{1}{2}(-x)^3-4$

Example: Write the equation of the function that is graphed after the following transformations are applied in order to the graph of $h(x)=\sqrt{x}$.

1. Vertical stretch by a factor of 3
2. Move left 5 units
3. Reflect across the y-axis

$f(x)=3\sqrt{-x+5}$

Example: Write the equation of the function that is graphed after the following transformations are applied in order to the graph of $f(x)=|x|$.

1. Horizontal compression by a factor of 1/2
2. Move up 6 units
3. Reflect across the x-axis

$f(x)=-\left|\frac{1}{2}x\right|+6$

Summary of Graphing Transformations:

To Graph:	Draw the Graph of $y=f(x)$ and:	Functional Change to $y=f(x)$:
Reflection About the x-axis		
$y=-f(x)$	Reflect the graph of f about the x-axis.	Multiply $f(x)$ by -1.
Reflection About the y-axis		
$y=f(-x)$	Reflect the graph of f about the y-axis.	Replace x by $-x$.
Vertical Stretches & Compressions (shrink)		
$y=af(x)$, $a>0$	Multiply each y-coordinate of $y=f(x)$ by a. This stretches the graph of f vertically if $a>1$. This compresses the graph of f vertically if $0<a<1$.	Multiply $f(x)$ by a.
Horizontal Stretches & Compressions		
$y=f(bx)$, $b>0$	Divide each x-coordinate of $y=f(x)$ by b. This stretches the graph of f horizontally if $0<b<1$. This compresses the graph of f horizontally if $b>1$.	Replace x by bx.
Vertical Shifts		
$y=f(x)+k$, $k>0$	Raise the graph of f by k units.	Add k to $f(x)$.
$y=f(x)-k$, $k>0$	Lower the graph of f by k units.	Subtract k from $f(x)$.
Horizontal Shifts		
$y=f(x-h)$, $h>0$	Shift the graph of f to the right by h units.	Replace x by $x-h$.
$y=f(x+h)$, $h>0$	Shift the graph of f to the left by h units.	Replace x by $x+h$.

$a\cdot f(b\cdot x+h)+k$

p65: 13, 21, 23, 25, 27, 28,
33, 35, 39

p.72: 9, 11-20, 21, 23, 29, 31,
37, 41, 43, 45, 47, 49, 51,
53, 56, 57