

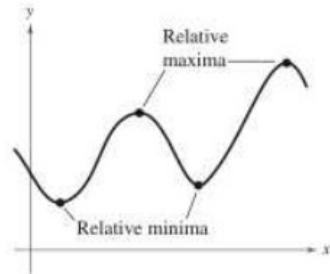
### Definitions of Relative Minimum and Relative Maximum

A function value  $f(a)$  is called a **relative minimum** of  $f$  when there exists an interval  $(x_1, x_2)$  that contains  $a$  such that

$$x_1 < x < x_2 \text{ implies } f(a) \leq f(x).$$

A function value  $f(a)$  is called a **relative maximum** of  $f$  when there exists an interval  $(x_1, x_2)$  that contains  $a$  such that

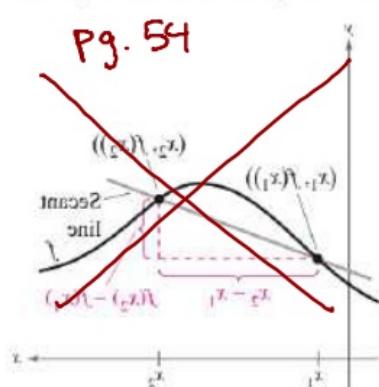
$$x_1 < x < x_2 \text{ implies } f(a) \geq f(x).$$



$$f(x) = -4x^2 - 7x + 3$$

( -0.875, 6.0625 ) increasing  $(-\infty, -0.875)$   
decreasing  $(-0.875, \infty)$   
never constant

### Average Rate of Change of a Function



$$f(x) = x^2 + 2x$$

a)  $x_1 = 3 \rightarrow x_2 = -2$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$f(x_1) = 3^2 + 2(3) = 9 + 6 = 15 \quad \frac{0 - 15}{-2 - 3} = \frac{-15}{-5} = 3$$

$$f(x_2) = (-2)^2 + 2(-2) = 4 - 4 = 0$$

### Even and Odd Functions

A function is said to be **even** when its graph is symmetric with respect to the y-axis and **odd** when its graph is symmetric with respect to the origin. They symmetry tests from Section 1.2 yield the following tests for even and odd functions.

### Tests for Even and Odd Functions

A function  $y = f(x)$  is **even** when, for each  $x$  in the domain of  $f$ ,  $f(-x) = f(x)$ .

A function  $y = f(x)$  is **odd** when, for each  $x$  in the domain of  $f$ ,  $f(-x) = -f(x)$ .

$$y = x^2$$

$$y = x^3$$

$(-2, 3) \quad (2, 3)$

$(-2, -3) \quad (2, -3)$

## Section 1.6 A Library of Parent Functions

Constant Function

$$y = a$$

- Domain:  $\mathbb{R}$

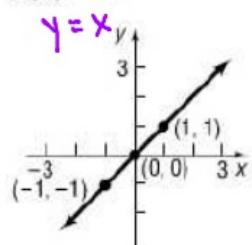
- Range:  $a$

- horizontal line through  $a$ .

**Identity Function**

$$f(x) = x$$

(linear)



- Domain:  $\mathbb{R}$

- Range:  $\mathbb{R}$

- Line with slope of  $m = 1$

- Intercept:  $(0,0)$

- Odd Function

- Increasing on  $(-\infty, \infty)$

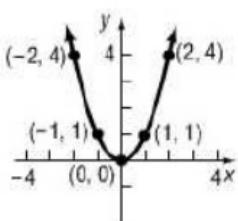
- Bisects Quadrants I and III

- Key Points:  $(-2, -2)$ ,  $(-1, -1)$ ,  $(0, 0)$ ,  $(1, 1)$ ,  $(2, 2)$

**Square Function**

$$f(x) = x^2$$

(Quadratic)



- Domain:  $\mathbb{R}$

- Range:  $[0, \infty)$

- Parabola

- Intercept:  $(0,0)$

- Even Function

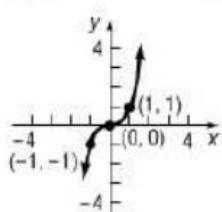
- Decreasing on  $(-\infty, 0)$  and increasing on  $(0, \infty)$

- Key Points:  $(-2, 4)$ ,  $(-1, 1)$ ,  $(0, 0)$ ,  $(1, 1)$ ,  $(2, 4)$

**Cube Function**

$$f(x) = x^3$$

(cubic)



- Domain:  $\mathbb{R}$

- Range:  $\mathbb{R}$

- Intercept:  $(0,0)$

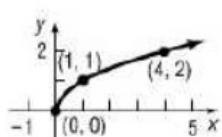
- Odd Function

- Increasing on  $(-\infty, \infty)$

- Key Points:  $(-2, -8)$ ,  $(-1, -1)$ ,  $(0, 0)$ ,  $(1, 1)$ ,  $(2, 8)$

**Square Root Function**

$$f(x) = \sqrt{x}$$



- Domain:  $[0, \infty)$

- Range:  $[0, \infty)$

- Intercept:  $(0,0)$

- Neither even nor odd

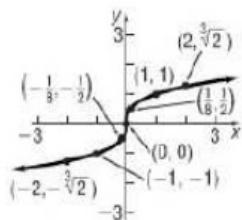
- Increasing on  $[0, \infty)$

- Minimum value of 0 at  $x = 0$

- Key Points:  $(0, 0)$ ,  $(1, 1)$ ,  $(4, 2)$ ,  $(9, 3)$

### Cube Root Function

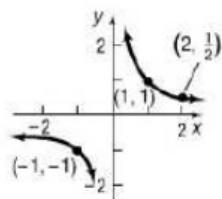
$$f(x) = \sqrt[3]{x}$$



- Domain:  $\mathbb{R}$
- Range:  $\mathbb{R}$
- Intercept:  $(0,0)$
- Odd Function
- Increasing on  $(-\infty, \infty)$
- Key Points:  $(-8, -2)$ ,  $(-1, -1)$ ,  $(0, 0)$ ,  $(1, 1)$ ,  $(8, 2)$

### Reciprocal Function

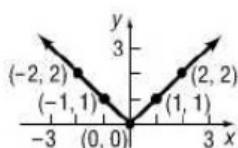
$$f(x) = \frac{1}{x}$$



- Domain:  $(-\infty, 0) \cup (0, \infty)$
- Range:  $(-\infty, 0) \cup (0, \infty)$
- No Intercepts
- Odd Function
- Decreasing on  $(-\infty, 0)$  and  $(0, \infty)$
- Key Points:  $(-2, -\frac{1}{2})$ ,  $(-1, -1)$ ,  $(-\frac{1}{2}, -2)$ ,  $(\frac{1}{2}, 2)$ ,  $(1, 1)$ ,  $(2, \frac{1}{2})$

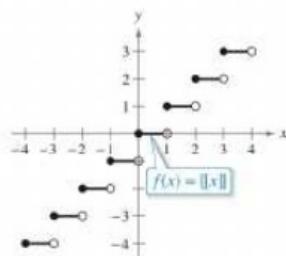
### Absolute Value Function

$$f(x) = |x|$$



- Domain:  $\mathbb{R}$
- Range:  $[0, \infty)$
- Intercept:  $(0,0)$
- Even Function
- Decreasing on  $(-\infty, 0)$  and increasing on  $(0, \infty)$
- Minimum value of 0 at  $x = 0$
- Key Points:  $(-2, 2)$ ,  $(-1, 1)$ ,  $(0, 0)$ ,  $(1, 1)$ ,  $(2, 2)$

### Step Function (greatest integer)



- The domain of the function is the set of all real numbers.  $\mathbb{R}$
- The range of the function is the set of all integers.
- The graph has a y-intercept at  $(0, 0)$  and x-intercepts in the interval  $[0, 1]$ .
- The graph is constant between each pair of consecutive integer values of  $x$ .
- The graph jumps vertically one unit at each integer value of  $x$ .

$$\lfloor \text{ } \rfloor$$

$$\lceil 2.3 \rceil = 2$$

$$\lfloor 4 \rfloor = 4$$

$$\lceil 13.7 \rceil = 13$$

$$\lceil -1.2 \rceil = -1$$

$$\leftarrow \frac{1}{-2} -1 0 \rightarrow$$