Section 1.4 Functions (continued)

Finding Values for Which $f(x) = g(x)$.

Example: $f(x) = x^2 + 6x - 24$ and $g(x) = 4x - x^2$

$x^2 + 6x - 24 = 4x - x^2$

$+x^2 - 4x \quad -4x + x^2$

$2x^2 + 2x - 24 = 0$

$2(x^2 + x - 12) = 0$

$2(x+4)(x-3) = 0$

$\frac{x}{-12} \quad \frac{+}{1}$

4, -3

$16-24-24 \quad -16-16$

$-32 = -32$

$2 \neq 0$

$x+4=0$ or $x-3=0$

$-4 \; -4 \qquad +3 \; +3$

$\boxed{x=-4,\ 3}$

$x(x+4)$

$x=0, -4$

Domain of a Function

The domain of a function *f(x)* is the set of all real numbers for which the expression is defined, a set of all inputs *x*.

- If the function is listed in a table or as a set of ordered pairs, the domain is the set of all the first coordinates.
- If the function is described by a graph, the domain is the set of all *x*-coordinates of the points on the graph.
- If the function is described by an equation, the domain is the set of all real numbers for which *f(x)* is a real number. Figure out if there any *x*-values that cause "problems" (zero in a denominator, square root of a negative, etc.) when you plug them into the function. If so, these numbers are not part of the domain.
- If the function is used in an application, the domain is the set of all numbers that make sense in the problem.

Tips for finding domain:

1. If the equation has fractions, exclude any numbers that give a zero in a denominator. $\frac{1}{x}$ $x \neq 0$
2. If the equation has an even root, exclude any numbers that cause the expression under the root to be negative. $\sqrt{x}$ $x \geq 0$

Examples: Find the domain for each function.

$f: \{(-2,-2), (-1,1), (0,3), (1,1), (2,2)\}$

x: $\{-2, -1, 0, 1, 2\}$

$f(x) = x^2 - 2$

$\mathbb{R}$ → all reals

$(-\infty, \infty)$

$f(x) = \frac{4x}{x^2 - 9}$

$x^2 - 9 \neq 0$

$(x+3)(x-3) \neq 0$

$x \neq -3, 3$

$(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

$f(x) = |x - 5|$

$(-\infty, \infty)$

Circumference of a cirlcle: $C = 2\pi r$

$C(r) = 2\pi r$

radius > 0

$r > 0$

$(0, \infty)$

$f(x) = \sqrt{x+2}$

$x + 2 \geq 0$

$-2 \quad -2$

$x \geq -2$

$[-2, \infty)$

$f(x) = \frac{1}{\sqrt{x+2}}$

$x + 2 > 0$

$x > -2$

$(-2, \infty)$

$f(x) = \sqrt{-5x+7}$

$-5x + 7 \geq 0$

$-7 \quad -7$

$\frac{-5x}{-5} \geq \frac{-7}{-5}$

$x \leq \frac{7}{5}$

Difference Quotients

Given

The expression $\frac{f(x+h)-f(x)}{h}, h \neq 0$ is called the difference quotient. It is the basis of many of the ideas in calculus. One thing that makes finding the difference quotient of a function easier is to look at all the parts of the expression separately:

Examples: Find the difference quotient of *f(x)*, be sure to simplify.

Given $f(x) = 2x - 5$

$f(x+h) = 2(x+h) - 5$

$= 2x + 2h - 5$

$\frac{(2x+2h-5) - (2x-5)}{h}$

$= \frac{\cancel{2x} + 2h \cancel{-5} \cancel{-2x} \cancel{+5}}{h}$

$= \frac{2h}{h}$

$= 2$

Given $f(x) = 5x^2 - x + 4$

$f(x+h) = 5(x+h)^2 - (x+h) + 4$

$= 5(x^2 + 2xh + h^2) - x - h + 4$

$= 5x^2 + 10xh + 5h^2 - x - h + 4$

$\frac{(\cancel{5x^2} + 10xh + 5h^2 \cancel{-x} - h \cancel{+4}) + (\cancel{-5x^2} \cancel{+x} \cancel{-4})}{h}$

$= \frac{10xh + 5h^2 - h}{h}$

$= \frac{\cancel{h}(10x + 5h - 1)}{\cancel{h}}$

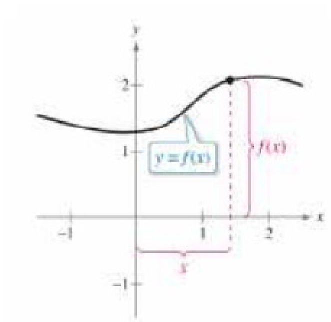
$= 10x + 5h - 1$

Section 1.5 Analyzing Graphs of Functions

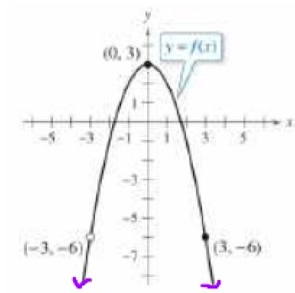
(x, y)

The **graph of a function** is the collection of ordered pairs $(x, f(x))$ such that x is in the domain of f.

$x =$ the directed distance from the y-axis
$y = f(x) =$ the directed distance from the x-axis



Example: Finding the Domain and Range of a Function.



Use the graph to find:

Domain $(-\infty,-3)\cup(-3,\infty)$

$f(0) = 3$

$f(3) = -6$

Range $(-\infty, 3]$

Find x for $f(x) = -6$

$\boxed{x=3}$

Zeros of a Function

If the graph of a function of x has an x-intercept at $(a,0)$, then a is a **zero** of the function.

The **zeros of a function** f of x are the x-values for which $f(x) = 0$.

Examples:

$f(x) = 2x^2 + 13x - 24$

	x	8
$2x$	$2x^2$	$16x$
-3	$-3x$	-24

$\times$: $2x \cdot -48$ $\quad$ $+$: 13

$16, -3$

$(2x-3)(x+8)=0$

$2x-3=0 \qquad x+8=0$
$+3 \; +3 \qquad -8 \; -8$

$\frac{2x}{2} = \frac{3}{2}$

$\boxed{x = \frac{3}{2}, -8}$

Increasing, Decreasing, and Constant Funtions

Increasing, Decreasing, and Constant Functions

A function f is **increasing** on an interval when, for any x_1 and x_2 in the interval,

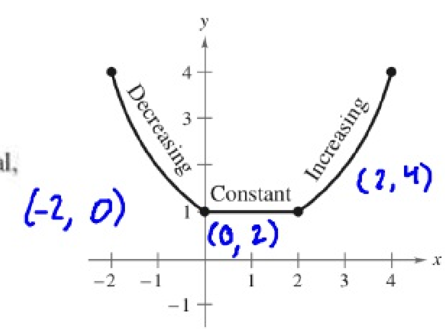
$x_1 < x_2$ implies $\overset{y_1}{f(x_1)} \overset{<}{<} \overset{y_2}{f(x_2)}$.

A function f is **decreasing** on an interval when, for any x_1 and x_2 in the interval,

$x_1 < x_2$ implies $\overset{y_1}{f(x_1)} \overset{>}{>} \overset{y_2}{f(x_2)}$.

A function f is **constant** on an interval when, for any x_1 and x_2 in the interval,

$f(x_1) = f(x_2)$.



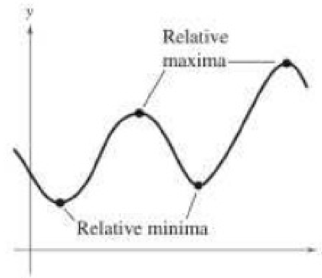
Definitions of Relative Minimum and Relative Maximum

A function value $f(a)$ is called a **relative minimum** of f when there exists an interval (x_1, x_2) that contains a such that

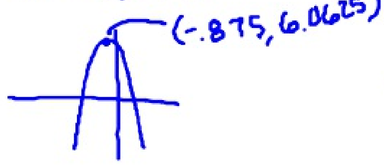
$$x_1 < x < x_2 \quad \text{implies} \quad f(a) \le f(x).$$

A function value $f(a)$ is called a **relative maximum** of f when there exists an interval (x_1, x_2) that contains a such that

$$x_1 < x < x_2 \quad \text{implies} \quad f(a) \ge f(x).$$



$f(x) = -4x^2 - 7x + 3$

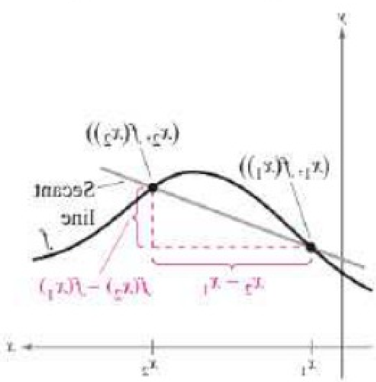


increasing $(-\infty, -.875)$

decreasing $(-.875, \infty)$

never constant

Average Rate of Change of a Function



Even and Odd Functions

A function is said to be **even** when its graph is symmetric with respect to the y-axis and **odd** when its graph is symmetric with respect to the origin. They symmetry tests from Section 1.2 yield the following tests for even and odd functions.

Tests for Even and Odd Functions

A function $y = f(x)$ is **even** when, for each x in the domain of f, $f(-x) = f(x)$.

A function $y = f(x)$ is **odd** when, for each x in the domain of f, $f(-x) = -f(x)$.