Examples of **polynomial functions**:

$f(x) = ax + b$ linear

$f(x) = c$ constant

$f(x) = x^2$ quadratic

$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$ is a **polynomial function of *x* with degree *n*.**

Definition of Quadratic Function:

Let *a*, *b*, and *c*, be real numbers with $a \neq 0$. The function $f(x) = ax^2 + bx + c$ is called a **quadratic function.**

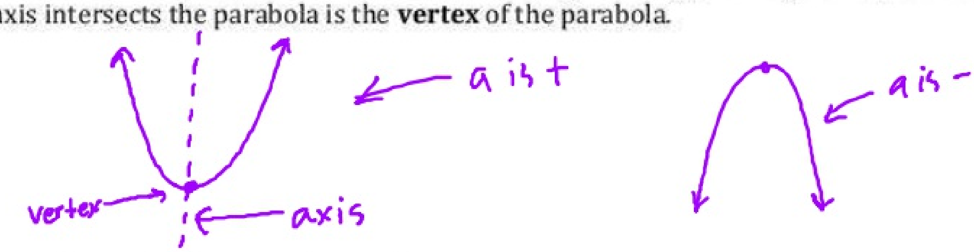
$f(x)=x^2+6x+2$ general form

$g(x)=2(x+1)^2-3$ vertex form / standard

$m(x)=(x-2)(x+1)$ factored form

The graph of a quadratic function is called a **parabola**.

All parabolas are symmetric with respect to a line called the **axis of symmetry** (or **axis**) of the parabola. The point where the axis intersects the parabola is the **vertex** of the parabola.



Graphing a Quadratic Function Using Transformations

1. Begin with the parent function $f(x) = x^2$.
2. Rewrite the function in vertex form $f(x) = a(x-h)^2 + k$ by completing the square.
3. Transform with the following:

a: If a is positive, the graph opens up. The y-coordinate of the vertex is a minimum value.
If a is negative, the graph opens down. The y-coordinate of the vertex is a maximum value.
If $|a| > 1$, the graph is narrower than the graph of $f(x) = x^2$. vert. stretch
If $|a| < 1$, the graph is wider than the graph of $f(x) = x^2$. vert. shrink

h: h controls the horizontal shift (left and right). $+h$: left $-h$: right

k: k controls the vertical shift (up and down). $+k$: up $-k$: down

Vertex: (h, k) Axis of Symmetry: $x = h$

Example: Sketch the graph of each quadratic function and compare it with the graph of $y = x^2$.

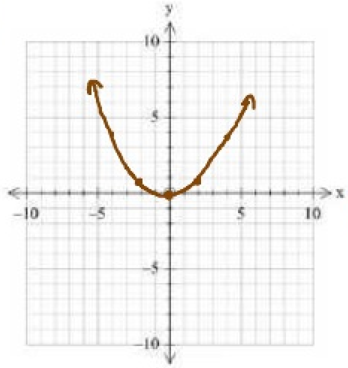
a. $f(x) = \frac{1}{4}x^2$

vertex: (0,0)

axis: x=0

wider than x^2

x	y
2	$\frac{1}{4}(2)^2 = 1$
4	$\frac{1}{4}(4)^2 = 4$



b. $k(x) = -4x^2+2$

vertex: (0,2)

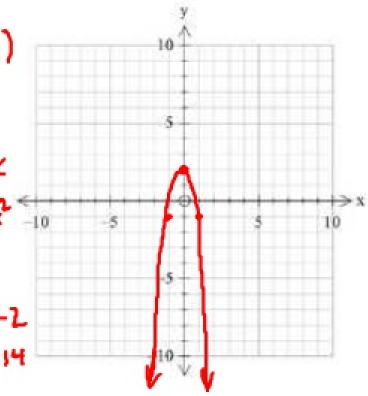
axis: x=0

reflect over x

narrower than x^2

up 2

x	y
1	$-4(1)^2+2=-2$
2	$-4(2)^2+2=-14$



$(x-2)^2$

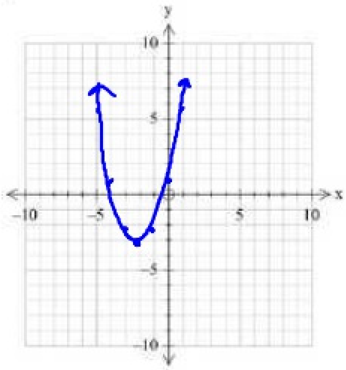
c. $f(x) = (x+2)^2 - 3$

Vertex: (-2,-3)

axis: x=-2

left 2

down 3



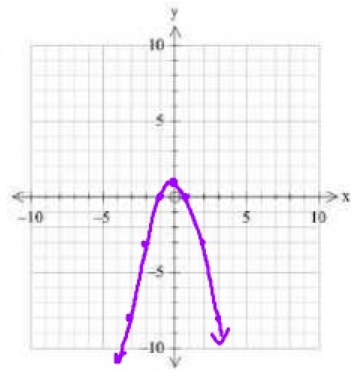
d. $f(x) = -x^2 + 1$

vertex: (0,1)

axis x=0

reflect over x

up 1



Using Standard Form of a Quadratic Function

Completing the Square: Figuring out what constant to add to a binomial of the form $x^2 + bx$ to make it into a perfect square trinomial, then writing the result in factored form.

Completing the Square for the Binomial $x^2 + bx$

1. Divide the coefficient of the x-term by 2. $\left(\text{Find } \frac{b}{2}\right)$.
2. Square the answer from step 1. $\left(\text{Find } \left(\frac{b}{2}\right)^2\right)$.
3. Add the result of step 2 to the binomial.
4. Rewrite as a perfect square: $\left(x+\frac{b}{2}\right)^2$.

Example: Add the proper constant to the binomial to make it into a perfect square trinomial. Then factor the trinomial.

x^2+12x+___	x^2-7x+___
$\frac{12}{2}=6$ $(6)^2=36$ $x^2+12x+36$ $(x+6)^2$	$-\frac{7}{2}$ $\left(-\frac{7}{2}\right)^2=\frac{49}{4}$ $x^2-7x+\frac{49}{4}$ $\left(x-\frac{7}{2}\right)^2$

Examples: Add the proper constant to each binomial to make it into a perfect square trinomial. Then factor the trinomial.

a) $x^2+16x+\underline{64}=(x\underline{+8})^2$ $\frac{16}{2}=8$ 8^2

d) $x^2-3x+\underline{\frac{9}{4}}=(x\underline{-\frac{3}{2}})^2$ $-\frac{3}{2}$ $(-\frac{3}{2})^2$

e) $x^2+\frac{4}{3}x+\underline{\frac{4}{9}}=(x\underline{+\frac{2}{3}})^2$ $\frac{\frac{4}{3}}{2}$ $\frac{4}{3}\cdot\frac{1}{2}=\frac{2}{3}$ $(\frac{2}{3})^2$

Writing $f(x)=(ax^2+bx)+c$ in Vertex Form

1. Group ax^2 and bx together in parentheses.
2. If $a\neq 1$, factor out a from ax^2+bx. Include a negative if the quadratic term is negative.
3. Complete the square (divide b by 2 and square the result). Add the answer inside the parentheses. Keep the equation balanced by adding or subtracting outside the parentheses. (You are adding 0 to one side of the equation.)
4. Write the expression inside the parentheses as a perfect square.

Examples: Write each equation in vertex form. Then find the vertex.

a) $f(x)=(x^2-8x)-5$
$(x^2-8x+\underline{16})-5-\underline{16}$
$\boxed{f(x)=(x-4)^2-21}$

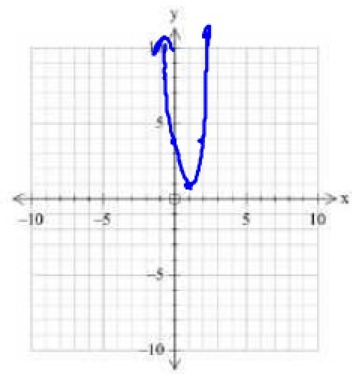
b) $f(x)=(3x^2+6x)+1$
$3(x^2+2x+\underline{1})+1-\underline{3}$
$\boxed{f(x)=3(x+1)^2-2}$

c) $y=(-x^2+4x)-3$
$-(x^2-4x+\underline{4})-3+\underline{+4}$
$\boxed{y=-(x-2)^2+1}$

Using Standard Form to Graph a Parabola
Example: Sketch the graph of $f(x)=3x^2-6x+4$.
Identify the vertex and the axis of the parabola.

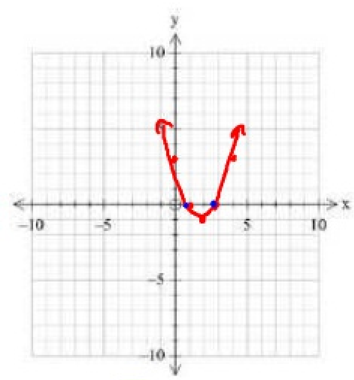
$f(x)=(3x^2-6x)+4$
$=3(x^2-2x+\underline{1})+4-\underline{3}$
$=3(x-1)^2+1$

vertex: $(1,1)$
axis: $x=1$



Finding the Vertex and x- intercepts of a Parabola
Example: Sketch the graph $f(x)=(x^2-4x)+3$.
Identify the vertex and x-intercepts. (To find x-intercepts, you may factor or use the Quadratic Formula).

$f(x)=(x^2-4x+\underline{4})+3-\underline{4}$
$f(x)=(x-2)^2-1$
$\boxed{\text{vertex: }(2,-1)}$



for x-intercepts factor $f(x)$, then set $=0$

$f(x)=x^2-4x+3$
$=(x-3)(x-1)=0$

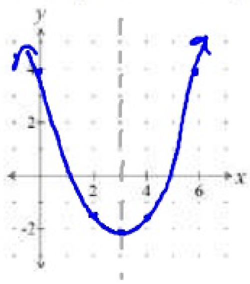
$x-3=0$ $x=3$
$x-1=0$ $x=1$

$\boxed{\begin{matrix}(3,0)\\(1,0)\end{matrix}}$

Writing a Quadratic Equation when You Know the Vertex and Another Point

1. Use vertex form: $y = a(x-h)^2 + k$
2. Plug in the vertex for h and k.
3. Plug in the other point for x and y (or $f(x)$).
4. Simplify and solve for a. (Don't forget order of operations.)
5. Write your final answer by plugging a, h, and k back into vertex form.

Example: Find the quadratic function whose vertex is $(3,-2)$ and whose y-intercept is 4. Graph the function.



$y = a(x-h)^2 + k$ $\quad (0,4)$

$y = a(x-3)^2 - 2$

$4 = a(0-3)^2 - 2$

$4 = a(-3)^2 - 2$
$+2 \qquad +2$

$6 = 9a$

$a = \frac{6}{9} = \frac{2}{3}$

$\boxed{f(x) = \frac{2}{3}(x-3)^2 - 2}$

or $y =$

Example: Write the standard form of the equation of the parabola whose vertex is (-4,11) and that passes through the point (-6,15).

h k

x y

$y = a(x+4)^2 + 11$

$15 = a(-6+4)^2 + 11$
$-11 \qquad -11$

$4 = a(-2)^2$

$4 = a(4)$

$a = 1$

$y = 1(x+4)^2 + 11$

$\boxed{y = (x+4)^2 + 11}$

Finding the Minimum and Maximum Values

By completing the square, we can rewrite $f(x) = ax^2 + bx + c$ as $f(x) = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$.

This gives us a quick way to find the vertex when the equation is in standard form:

- The x-coordinate of the vertex is $-\frac{b}{2a}$.
- To find the y-coordinate, plug the x-coordinate into the original equation.

$y = x^2 + 6x - 12$

$x: -\frac{6}{2(1)} = -3$

$y = (-3)^2 + 6(-3) - 12$
$= 9 - 18 - 12$
$= -21$

vertex
$(-3, -21)$

Vertex : $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$ **Axis of Symmetry :** The line $x = -\frac{b}{2a}$

Parabola opens up if $a > 0$; the vertex is a minimum point.
Parabola opens down if $a < 0$; the vertex is a maximum point.

	Standard Form	Vertex Form	Factored Form
Equation	$y = ax^2 + bx + c$	$y = a(x-h)^2 + k$	$y = a(x-p)(x-q)$
Vertex	Complete the square and write in vertex form. -or- $x = \frac{-b}{2a}$ Plug the x-coordinate into the equation to get the y-coordinate.	(h, k)	Find average of p and q. $x = \frac{p+q}{2}$ (The x-coordinate of the vertex is at the midpoint of the x-intercepts.) Plug the x-coordinate into the equation to get the y-coordinate.
y-intercept	c (Replace x with zero. Solve for y.)	Replace x with zero. Solve for y.	Replace x with zero. Solve for y.
x-intercepts (roots, zeros, solutions)	Replace y with zero. Solve for x by factoring or quadratic formula.	Replace y with zero. Solve for x by isolating the perfect square and using the square root principle. (Don't forget the $\pm$.)	p and q (Replace y with zero. Solve for x using the zero product property.)

For all forms:

Direction of Opening	Up if a is positive Down if a is negative
Vertical Stretch	a
Counting Pattern (Shortcut)	Start at the vertex. Find more points by counting: $\leftrightarrow 1$, $\uparrow a$ $\leftrightarrow 1$, $\uparrow 3a$ $\leftrightarrow 1$, $\uparrow 5a$ $\leftrightarrow 1$, $\uparrow 7a$, etc. (If a is negative, move down instead of up.)

p. 120: 7-12, 15, 17, 21, 23, 29, 37, 39, 44, 45, 47, 49, 51, 57, 77, 81